Dynamic adjustment of size profiles

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Declaration

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Abstract

The effect of dynamic size profile adjustments on total sales are analysed for a prominent South African fashion retailer. Stock arriving at the distribution centre (DC) from factories are allocated to stores via a push system assuming demand is deterministic, where the retailer finalises allocation decisions on a central level for all stores. Paramount to store allocation decisions are size profiles, which partition a fixed quantity of company stock available at the DC into smaller, ideal size-mix allocations for each store. The retailer derives size profiles from historical sales data, clustering stores with similar historic sales properties together. Each cluster receives a size profile reflective of the expected spread of sales amongst sizes, expressed as a percentage per size. Currently, size profiles remain static throughout the season, translating into inefficient stock allocations based on expected sales identified (only) from historic sales data.

In an attempt to improve stock allocation efficiency, most recent sales data made available are incorporated into the decision making process when finalising allocations by dynamically adjusting size profiles throughout the season. To quantify the effect of dynamic size profile adjustments, sales of a prominent South African fashion retailer are simulated for a season. Verification and validation of a simulation model, built to incorporate dynamic size profile adjustments concludes sales output is a sufficiently close representation of the real system.

The simulation model is applied to two summer and two winter products, resulting in four simulation models. Analysis of product sales simulation with dynamic size profile adjustment, record a combined average increase in total sales of 3.11% for summer products and 2.72% for winter products, compared to static size profile sales. Fundamental to the success of dynamic size profile adjustments is the choice of an appropriate weighting parameter, γ. Sensitivity analysis on value variation of γ was performed for each of the four simulation models. The main finding is that a chosen weighting parameter value is dataset specific and retaining historical sales data is important in the dynamic adjustment of size profiles.

Opsomming

Die effek van 'n dinamiese aanpassing van grootte-profiele op die totale verkope van 'n bekende Suid-Afrikaanse kleinhandelaar word ondersoek. Vooraad wat vanaf fabrieke by die distribusiesentrum aankom, word aan winkels toegeken volgens 'n sentrale stootstelsel, waarin aangeneem word dat die aanvraag konstant en deterministies is. In hierdie toekenningsbesluite is die grootte-profiel belangrik om 'n vaste hoeveelheid voorraad op te deel vir al die winkels volgens daardie winkel se ideale grootte-mengsel. Die kleinhandelaar bepaal grootte-profiele deur winkels volgens historiese verkope saam te groepeer. Elke groep winkels kry dan 'n grootte-profiel wat die verwagte verspreiding van verkope oor die verskillende groottes weerspieël. Tans bly hierdie grootte-profiele staties gedurende 'n seisoen, wat kan lei tot swak toekenningsbesluite.

In 'n poging om die voorraadtoekenning te verbeter, word die jongste beskikbare verkoopsdata gebruik in die besluitnemingsproses deur die grootte-profiele dinamies aan te pas. 'n Simulasie wat die verkope vir 'n seisoen simuleer, is geprogrammeer om die effek van hierdie dinamiese aanpassing te kwantifiseer. Die simulasiemodel is geverifieer en gevalideer met die gevolgtrekking dat die gesimuleerde stelsel die werklike stelsel bevredigend naboots.

Die simulasiemodel word toegepas op twee winter- en twee somerprodukte wat resultate vir vier verskillende simulasies verskaf. 'n Ontleding van die resultate toon 'n gekombineerde toename in verkope van 3.11% vir die somerprodukte en 2.72% vir winterprodukte teenoor die statiese grootte-profiele. Die sukses van die dinamiese aanpassing berus op 'n gepaste keuse van die wegingsparameter, γ. Sensitiwiteitsanalise op die waarde van γ toon dat die beste waarde van γ afhanklik is van die onderliggende datastel en dat die behoud van historiese verkope data belangrik is in die dinamiese aanpassing van grootte-profiele.

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CHAPTER 1

Introduction

Contents

Market orientation has been recognised by both academics and practitioners as a core competency in increasing a retailer's competitiveness for almost 60 years [15, 19]. A retailer is defined as a person, retail shop or business that sells goods or products [28]. Christopher *et al.* [6] characterise a fashion good or product as having short life-cycles and seasonal demand volatility. Market orientation regarding the identification and response to changing customer demand during a product's selling season is of paramount importance to fashion retailers.

Traditional fashion retailers release products two to four times a year, usually coinciding with the seasons of summer, autumn, winter and spring. During a product's selling season, traditional fashion retailers' supply chains are immutable and long lead times of the distribution network are inherent. The success of a fashion retailer entails ensuring that a product mix containing the correct product types and correct quantities are available at retail stores to meet expected customer demand. Restricted by a rigid supply chain and distribution network, traditional fashion retailers are required to finalise product mix orders months before the product's selling season starts. Consequently the response to changing customer demand during a product's selling season is restricted to what has been ordered and is currently available.

This chapter contains a discussion of the supply chain and distribution network in the broader context of a fashion retailer. Thereafter, a description of planning and allocation processes at a unique fashion retailer are supplemented by specific explanations, illustrating the scope and relevance of the thesis.

1.1 Supply chain and distribution network of a fashion retailer

The supply chain of a fashion retailer encompasses several ordered stages, enabling seasonal end consumer demand for a particular product to be satisfied. The stages include sourcing raw materials, manufacturing the raw materials into finished products, and shipping the finished products to distribution centres (DCs), where sorting and transport to retail stores commence. The finished products available at retail stores may then be purchased by end consumers, concluding the stages within a fashion retailer's supply chain.

The distribution network of a fashion retailer comprises all of the shipping stages within the supply chain. Finished goods shipped to DCs, and sorted products transported to retail stores are stages within a distribution network (amongst others). Inventory shipped at each of these stages are designed to fulfil end consumer demand and decisions regarding the quantity are governed by two main processes, namely planning and allocation. A schematic in Figure 1.1 displays the relationship between planning and allocation processes and the extent of each on the distribution network.

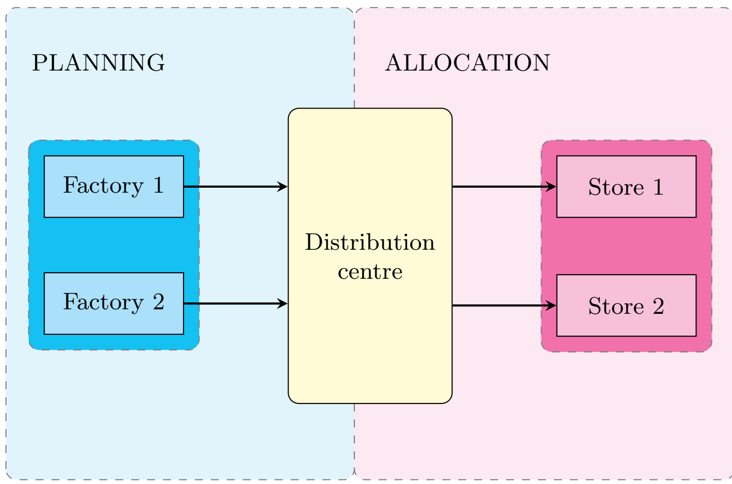

FIGURE 1.1: *A schematic representation of the distribution network of a typical retail supply chain.*

During the planning process the retailer determines product variety (how many products to order and offer to consumers), the breadth (how many types of the same product to order), and the depth (how many of each product type to order), creating what is known as assortment plans [33]. Assortment plans aim to ensure a correct product mix (of the correct products in the correct quantities) are available in anticipation of demand. The final phase in the planning process is to place assortment plans in the form of orders at factories. Raw materials are procured, which are manufactured into finished products by factories according to assortment plans. The distribution network ships finished product inventory to DCs, where it is sorted, stored and transported to retail stores completing the stages of a retailer's supply chain.

The amount of inventory each store receives is determined by allocation decisions made when stock arrives at the DC. The allocation process considers the amount of stock available at the DC, replenishment information depending on the system of allocation and stock constraints unique to each retailer. Allocation is driven by a push system or, the more common, pull system. In a pull system demand is assumed to be a random variable, allocation decisions are decentralised and reactive to information received from store managers [31]. Store managers request inventory based on their specific store replenishment needs, thereby "pulling" stock from the DC to satisfy store demand. Conversely allocation decisions made in a push system are based on anticipated demand, estimated by the retailer during the planning process; and made on a central level, for all stores. A centralised approach of allocation allows analysis of relevant information on a global level, for all stores.

The retailer considered in this study is the largest single brand retailer in Africa and sells amongst other things, clothing and footwear [30]. This retailer will be referred to as the "Retailer". The Retailer operates more than 2 200 stores in Southern Africa, sending an estimated 750 million products to its stores yearly [30]. The Retailer supplies two types of products to consumers, namely Type A and Type B products. Non-seasonal products, such as underwear or socks have a fairly constant demand over the whole year and are categorised as Type A products. This study is concerned with seasonal, fashion items available in either summer or winter. Fashion items are categorised as Type B products, where seasonal demand and short life-cycles are recorded. The planning and allocation process for Type B products are presented in §1.1.1 and §1.1.2, respectively.

1.1.1 The Retailer's planning process

Decisions made during the planning process influence the distribution network from when orders are placed until finished products arrive at the distribution centre. The planning process of the Retailer is done at a central level, by specialist planners in each department, for all stores. In the case of fashion/Type B products, the objective is to develop an assortment plan that will maximise sales and profit for a specified period of time – usually a season, such as summer or winter. Planners use historical sales data to achieve this objective. Planners infer demand which guide decisions regarding assortment plans (product variety, breadth and depth). The exact methodology followed by the Retailer during the planning process is not explicitly known by this study and demand is thus inferred from historical sales data. The Retailer expands product assortment to include decisions about how many different sizes of the product to offer end consumers and how many units of each size to order for the company, creating size-mix assortments.

Traditional fashion retailers such as this Retailer typically outsource manufacturing of products to factories, usually located in the Far East resulting in long lead times between planning and allocation processes. The Retailer places size-mix assortment orders at factories approximately 6–10 months before finished product inventory arrives at the DC, consequently restricting flexibility of the supply chain during the selling season and fixing the total inventory quantity of a product in the distribution network.

Size-mix assortment planning

A basic schematic of fashion product classification is presented in Figure 1.2, which assists in visualising planning decisions made by the Retailer regarding size-mix assortments. In each layer, moving from top to bottom Figure 1.2 illustrates decisions made in the planning process that finally result in a product size-mix assortment.

Only one size-mix assortment is expanded in this example. Other products offered by the Retailer follow the same methodology and structure presented in this example. Size-mix assortments are made by planners in the `Boys` department, where historical sales data is used to infer demand.

The first decision layer is regarding product variety, that is the number of products to order from factories and offer consumers during the selling season. For example, the product variety chosen for the boys department in Figure 1.2 is two products – `trousers` and `shirts`. The second decision layer is regarding product breadth, the number of product subclasses to offer consumers within each product variety. Expanding on boys shirts, specialist planners decide to offer two subclasses – `casual vests` and `short sleeved t-shirts`. Product depth is the third decision layer to be made and entails choosing the number of product styles of each subclass to

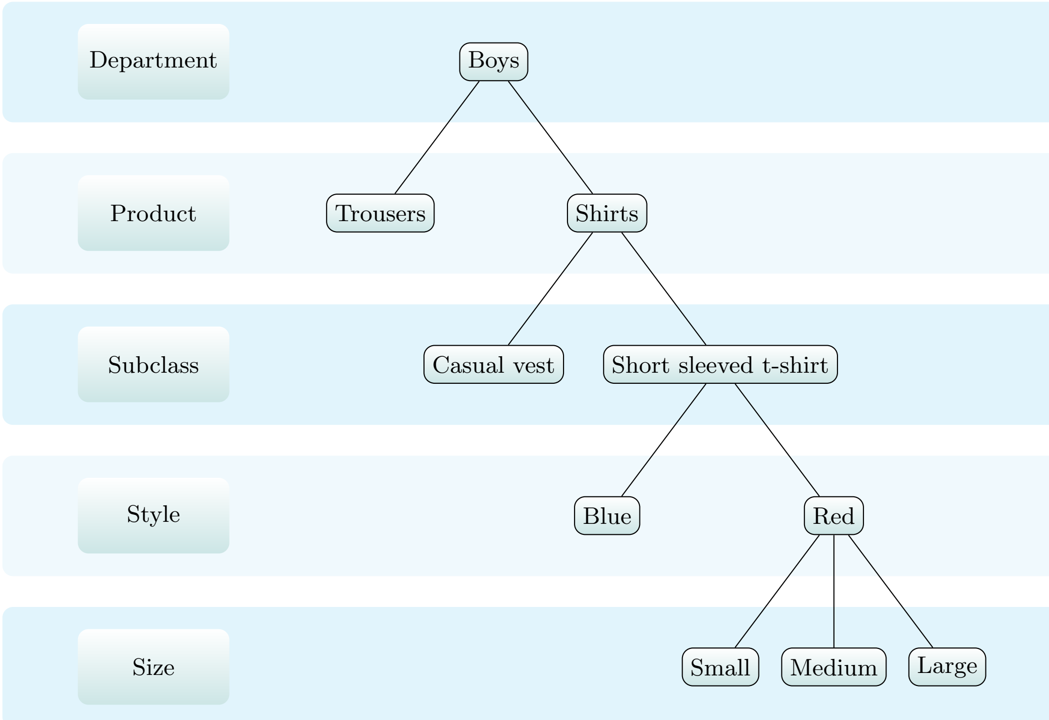

FIGURE 1.2: *An illustrative example of product classification according to subclasses, styles and then sizes.*

offer consumers. In this example planners decide a depth of two product styles is acceptable for `boys t-shirts`, offering one `blue` style and one `red` style of short sleeved t-shirt for boys. The final decision layer is especially critical in the fashion industry and determines how many sizes of the product to include in the offer to end consumers, and the quantity to order from factories for each size offered. Planners decide, for this example, that three sizes – `Small`, `Medium` and `Large` must be available in each store receiving the product assortment. The amount to order for each small, medium and large size of red short sleeved t-shirts is determined using historical sales data of similar products. Orders of each size are placed at factories for the company as a whole (*i.e.* all stores).

1.1.2 The Retailer's allocation process

Decisions made during the allocation process influence the quantity of stock within the distribution network from when finished products arrive at the DC until stock is available at stores. The allocation process is responsible for finalising the quantity of stock received by retail stores for each product ordered from factories. The aim is to send stock to stores in a quantity that will minimise shortages (due to a lack of stock) and surpluses (as a result of sending too much stock). A schematic representation of the distribution network specific to the Retailer is presented in Figure 1.3. Orders placed at factories take 6–10 months until finished products are delivered at the DC. The allocation process lasts for approximately 2–3 weeks and is initiated once finished products arrive at the DC, ending when stores receive stock inflow. The Retailer makes allocation decisions at a central level, classifying the allocation process as a push system. A push system relies on anticipated demand when making allocation decisions.

In the planning phase, specialist planners create size-mix assortments as described in §1.1.1,

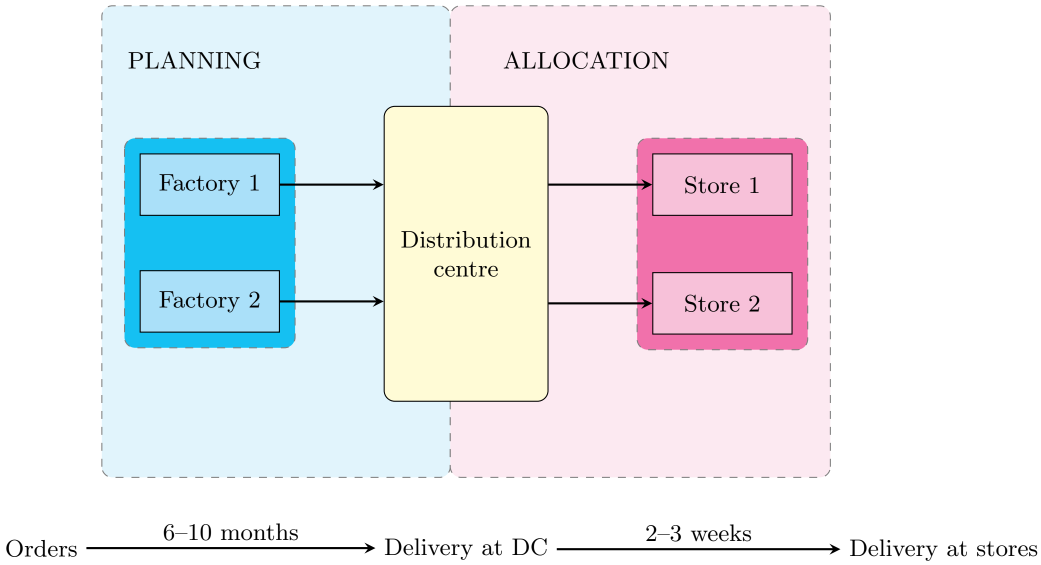

FIGURE 1.3: *A schematic representation of the distribution network for the Retailer.*

which define the range of sizes to offer end consumers and the quantity of each size to order from factories, for an assortment plan. Stock available at the DC is manufactured at factories according to these size-mix assortments which arrive throughout the selling season to ensure stores receive fresh stock of the product, in the form of styles. Each style of a product has an associated size-mix assortment. The quantity ordered of each size in a size-mix assortment reflects what the Retailer calls a "company profile" which is determined using historical sales data of the company (all stores). As size-mix assortment orders are placed at factories months before finished products arrive at the DC, the amount of stock available for allocation in each size is fixed. The allocation process is tasked with breaking down this fixed company size-mix, available at the DC, into smaller size-mixes for each store.

The Retailer's allocation process considers the amount of stock available at the DC, anticipated demand and stock constraints, which ensure all stores receive at least a minimum and no more than a maximum allocation. Anticipated demand is estimated from historical sales data by the Retailer during the planning process.

Preliminary allocations are made based on stores anticipated demand and indicate the quantity each store should be allocated for the style. The sum of preliminary allocation for all stores is equivalent to total stock of the fixed company size-mix available at the DC.

Anticipated demand on a size level for a store is made during the planning process by grouping stores with similar historical sales properties together, forming a cluster. Each cluster receives an associated size profile. These size profiles reflect the expected spread of sales over sizes for the group of stores and is given as a percentage per size. Size profiles form the foundation of size-mix allocation decisions.

When making allocation decisions for the company, each stores preliminary allocation along with its size profile is used to calculate an ideal size-mix. Once each store's ideal size-mix has been calculated, the size-mix allocation process finalises store allocations. The allocation process aims to send stock to stores as close as possible to the calculated ideal size-mix for each store, while considering the amount of stock available at the DC and stock constraints.

Size-mix allocation

The size-mix allocation process is centred around a store's preliminary allocation and size profile. Preliminary allocation decisions are equivalent to the amount of stock that was ordered from factories. When stock arrives at the DC from factories, allocation decisions consider the amount of stock available along with preliminary allocation decisions. On the other hand, the size profile is determined using historical sales data and is representative of the expected spread of sales as a percentage per size, for a cluster of stores with similar historical sales properties.

Currently the Retailer's size-mix allocation process is centred around a static size profile, meaning a store's expected spread of sales as percentage per size is unchanged throughout the selling season and based on historic sales data. This results in a consistent percentage of stock allocation (inflow) per size to a store. The actual unit inflow per size at a store may vary, due to preliminary allocation decisions that have been calculated per style to satisfy seasonal demand changes on a store level as historically observed. The percentage contribution per size at a store, however, remains consistent during a season due to a size profile which is static throughout the season.

To illustrate the effect of static size profiles on the allocation process, Table 1.1 contains a numeric example of a fictional store, Store A's stock allocation and recorded sales performance for two successive styles, as a percentage per size. The quantity of stock allocated per size to a store, is a function of the store's preliminary allocation and size profile. The size profile (as %) is listed for each size offered in Store A. Based on historic sales, the cluster in which Store A is grouped expects 15% of total sales recorded in the store to arise from small units, 20% from medium units, 37% from large units and 27% from extra large units. In the first line of Table 1.1, a preliminary allocation of 40 units is planned for Style 1, resulting in an ideal size-mix of 6, 8, 15 and 11 units for each respective small, medium, large and extra large size at Store A. The allocation process considers available stock at the DC, the calculated ideal size-mix for all stores and stock constraints. Store A is allocated the calculated ideal size-mix in units.

		small	medium	large	extra large
	Size profile (%)	15	20	37	27
Style 1 = 40	Allocated units	6	8	15	11
	Sold units	6	6	10	11
Style 2 = 60	Allocated units	9	12	22	16
	Sold units	8	8	10	15

TABLE 1.1: *Example of fictional Store A's size profile, size-mix allocation and recorded sales for two successive styles sent in one season.*

At the time of Style 2's arrival in the DC from factories, the amount of stock sold in each size at every store to date has been recorded. The second line in Table 1.1 presents the recorded total sales per size thus far, at fictional Store A. Considering each size, all 6 available small units were sold, 2 medium and 5 large units were unsold, and all 11 available extra large units were sold.

According to Store A's preliminary allocation decision for Style 2, a total of 60 units are planned. The ideal size-mix for small, medium, large and extra large sizes is calculated once again using the store's size profile and Style 2's preliminary allocation. The allocation process is able to send the ideal size-mix, resulting in 9, 12, 22 and 16 units of stock inflow per small, medium, large and extra large size, respectively. In comparison to the number of units allocated for Style 1, each size receives a different quantity, however, the percentage per size allocated is the same and reflective of Store A's size profile.

Given current sales information per size at Store A, the actual/current spread of sales as a percentage of total units sold is determined. This provides an opportunity to compare the expected spread of sales per size profile (determined from historic sales) with the actual/current spread of sales recorded to date.

A comparison between expected and actual/current spread of sales is presented in Figure 1.4 (a), where the red solid line depicts the expected spread of sales for fictional Store A, and the blue dashed line depicts the actual/current spread of sales recorded to date. Both these lines present the spread of sales as a percentage per size. Considering the spread of current sales small units appear to be selling more than expected from historical sales. Medium and large units record fewer units sold to date than expected and extra large units record considerably more unit sales than expected.

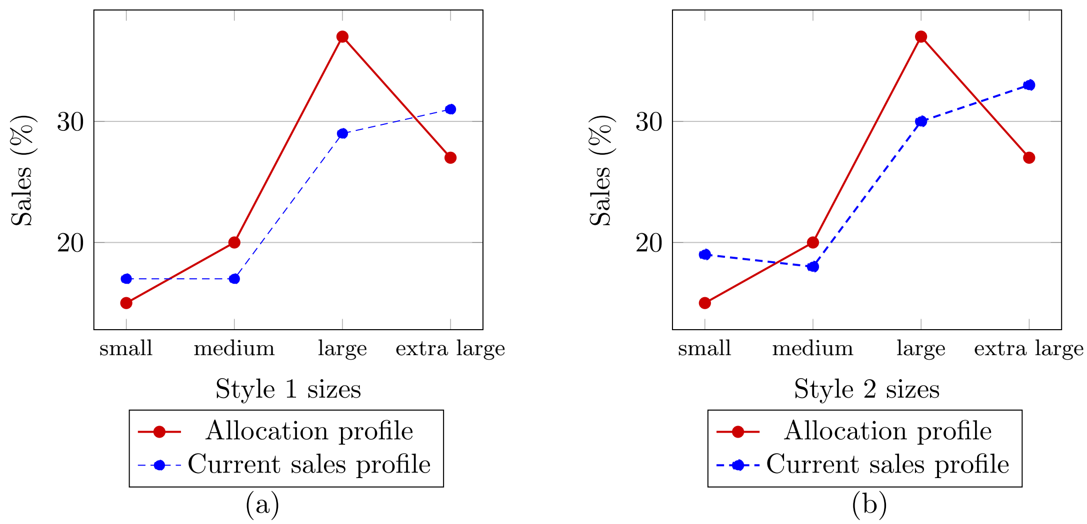

Figure 1.4: *Profile for fictional Store A over successive styles.*

For each successive style sent to stores during a season, cumulative sales per size at each store as recorded to date may be useful in assessing the current sales performance of a store. Cumulative sales indicate more reliable results when trying to identify patterns of changing customer demand. It is understood that if a store's size profile is an accurate representation of the expected spread of sales, the actual/current spread of sales at season's end would be close to the size profile made during the planning process. However, static size profiles are likely self-fulfilling prophesies of the recorded spread of sales, as retail stores are only able to sell what is available; and what is available is a result of the size-mix allocations.

Figure 1.4 (b) indicates the expected spread of sales (red solid line) for Style 2 and the current spread of sales (blue dashed line) recorded from the start of the season up to date at Store A. The red solid line is congruous with Style 1's allocation, due to the use of a static size profile. Analysis of the actual/current spread of sales (blue dashed line) in Figure 1.4 (b) is similar to the profile presented in Figure 1.4 (a). The similarity of actual/current profiles indicates sales for small, medium, large and extra large units at Store A persist in differing from expected sales, determined using historic sales. If fictional Store A's sales continue in this manner, a build-up of unsold medium and large units are likely to occur and, restricted by the availability of small and extra large units, an unmeasurable number of lost sales might likely occur throughout the season.

A continuous pattern of differing sales in sizes at a store is an indication of changing customer demand. The highly competitive and customer centric industry of fashion retailing should propel adaptive decision making during the allocation process. However, for many traditional retailers, decisions made during the allocation process are restricted to stock available at the DC, anticipated demand estimated by the retailer during the planning process and stock constraints.

1.2 Problem statement

A potential result of misguided anticipated demand is stock build-up and stock shortages, which in-turn leads to lost sales and/or discounts. This thesis aims to improve anticipated demand by dynamically adjusting size profiles as current sales data becomes available throughout the selling season. The main objective is to analyse the effect of dynamic size profile adjustments on total sales for all stores and sizes within the company, as well as unique subsets of stores (and sizes).

1.3 Objectives

The problem stated in this thesis will be addressed by the following objectives:

1. Describe the problem of allocation adjustment decisions in relation to a traditional fashion retail supply chain and distribution network.
2. Describe existing literature on size-mix allocation and simulation as a method to measure model (dynamic size profile adjustment) effectiveness.
3. Collect, clean and validate relevant data to solve size-mix allocation decisions and to measure the effectiveness of dynamic size profile adjustments.
4. Describe a simulation model, all relevant input parameters generated and an existing size-mix allocation algorithm.
5. Develop and describe a size profile adjustment algorithm.
6. Test the validity and accuracy of the simulation model.
7. Use the simulation model to measure the effect of dynamic size profile adjustments.
8. Summarise findings from the study and make recommendations based on results. Discuss ideas for future research and provide a summary of the study.

1.4 Thesis layout

The remainder of this thesis will be structured as follows. Literature related to the study is discussed in Chapter 2. Data received from the Retailer are discussed in Chapter 3 along with the simulation model, a dynamic size profile adjustment algorithm and the size-mix allocation. Chapter 4 validates the simulation model for summer and winter products considered in this study. Results of dynamic size profile adjustments are provided in Chapter 5 for summer and winter products. Finally, in Chapter 6, a summary of findings are discussed, recommendations are made based on results and ideas for future research are provided, followed by a summary of the work completed in this study.

CHAPTER 2

Literature review

Contents

The distribution network of a fashion retailer has two underlying processes: the planning process and the allocation process. The planning process of the Retailer is performed 6–10 months before stock arrives at the DC from factories, thereafter the allocation process commences. The main contribution of this study is the dynamic adjustment of size profiles which occur once stock arrives at the DC from factories. Size profile adjustment thus transpires during the Retailer's size-mix allocation process.

Literature on the planning process is presented in § 2.1 and followed by the allocation process in § 2.2. This study uses simulation to measure the effectiveness of dynamic size profile adjustments. Simulation as a tool is discussed in §2.4 and subsequently the Retailer's simulation and related topics are discussed in § 2.5.

2.1 Planning

The planning process at a fashion retailer consists of assortment planning and placing orders for the company as a whole at factories. Assortment planning includes deciding how many and which products to include in the product line, how many and which styles for each product to buy, and how many and which product sizes to buy [33]. Orders reflecting assortment plans are placed at factories in due course for store distribution.

Fashion retailers are required to periodically update and adjust assortment plans due to several factors such as changes in seasons, fashion trends and customer buying behaviour. Long development, procurement, and production lead times are common to traditional fashion retailers, enforcing the planning process and all related decisions to be made months prior to the selling season [20]. Constrained by rigid and long design-to-shelf lead times, traditional fashion retailer assortment plans are updated annually or biannually when the season ends, in preparation for the following season.

The importance of responding to changing customer demand and ensuring products are available where they are desired is widely understood by fashion retailers. In recent years the rise of innovative “fast-fashion” firms, such as Zara (Mango, and World Co.) have changed the trajectory of assortment planning by implementing highly responsive and flexible supply chains that cut the design-to-shelf lead time down to a few weeks [4]. Fast-fashion retailers update assortment plans during the selling season, allowing them to react quickly to changing customer demand and fashion trends [20]. Highly responsive and flexible supply chains come at an extraordinary high cost that many fashion retailers such as the one considered in this study, cannot afford to incur. Leaving decisions in the allocation process as a potential area of optimisation for traditional fashion retailers.

2.2 Allocation

Due to heterogeneous nature of the market place, fashion retailers are required to tailor their assortments according to store demands [23]. During the planning process, ordering decisions for the company as a whole have been made and factories complete these orders accordingly, meaning the amount of stock available in the allocation process is fixed. Planning and allocation processes are related but given the nature of each process, problems that arise in each process are solved independently.

The general allocation problem has been well researched, and involves the allocation of stock to stores from a central warehouse or DC [40]. Two systems exist within the allocation process and the use of information (in the allocation process) distinguishes a pull system from a push system. Most fashion retailers such as Zara, make use of a pull system, where demand is assumed to be a random variable and allocation decisions are (localised) dependent on local information, in the form of store manager requests [31, 38]. The majority of literature available is on pull allocation (literature on local and central control have rarely intersected [12]).

Allocation decisions in a push system are based on anticipated demand and are made at a central level, using global information for all stores [31, 38]. Clark & Scarf [7] initiated the study of distribution systems under central control in 1960. In central control all information flows to one point, where all decisions are made [12]. The retailer in this study uses a push system, where anticipated demand is determined using historical sales data from previous seasons. Centralising allocation decisions assists in keeping expenses low (no manager salaries in all stores), ultimately benefiting the end consumer. A disadvantage of the push system arises from the absence of current sales data when finalising allocation decisions. Not incorporating current sales data means allocations reflect only historical sales and are not responsive to changing customer demand. A lack of consideration towards changing customer demand throughout the season could result in stock built-up, where actual demand is less than anticipated; or lost sales, from an underestimation of anticipated demand.

2.3 Size-mix allocation

The general allocation problem does not specifically consider allocation decisions for products consisting of different sizes. Furthermore, literature on size-mix allocation decisions within a push system are limited.

A study by Caro & Gallien [4, 5] formulated a mixed-integer programming problem to solve Zara's size-mix allocation problem, where total sales are maximised subject to stock constraints unique to the retailer. Inputs to the model include forecasts of future sales, inventory levels of each size in the warehouse and decisions about the size-mix made during the planning process. Forecasts are done using historical data and requests from store managers, categorising this study of size-mix allocation within a pull system due to store managers requests. Including historical data in the allocation process improved sales by 3 to 4%, compared to only considering store managers' requests [5].

The allocation process, no matter the system, aims to send stock to satisfy demand at stores. Thom [40] tested four size-mix allocation models developed for the Retailer. The aim was to improve the breakdown of a product's fixed company size-mix available at the DC, determined during the planning process into smaller size-mixes for each store. All four models aimed to send stock that would satisfy each store's anticipated demand per size, subject to the amount of stock available at the DC, stock constraints and bounds, restricting the number of units in each size that may be allocated to each store as specified by the Retailer. The bounds ensure all stores receive stock sufficient to cover anticipated demand, (preventing a situation where some stores are not sent enough stock at the benefit of other stores). Thom [40] found all four allocation methods to be approximately equally effective with no significant difference between them. A possible reason for this outcome is the unchanging percentage inflow amongst sizes a store receives throughout the season for a product, which is completely based on historical sales performance recorded in size profiles.

Messina [24] conducted a pilot study that addressed the same problem as the one considered in this thesis. Size profile adjustment enabled each store's expected spread of sales to reflect a combination of historic and current sales data. The study aimed to determine whether the addition of current sales data in the calculation of inflows would have an effect on total sales, shortages and surpluses at the end of the season. A sample of six stores (two small, two medium and two large) were chosen at random from a population of 1 297 stores. These stores all received the same product throughout the season and all relevant allocation information was available. Sales were generated weekly and size profiles were adjusted accordingly throughout the season. On average, sales increased by 4.04%, shortages decreased by 10.53% and surpluses decreased by 12.72% for these six stores, compared to static size profile sales, shortages and surpluses. The pilot study concluded that dynamic adjustment of size profiles has merit.

2.4 Simulation

Several methods to measure the effectiveness of models (techniques) exist in literature. Simulation is the most suitable method for this study, as other methods (such as analytical methods) limit experimentation across products. Real life tests are often too time consuming to implement. They are not equally comparable to one another as one store cannot implement multiple experiments in parallel. The possibility of human error is also inevitable in real life tests. Simulation is a technique used to imitate the operations of a real-world facility or process as it evolves over time and is a means for testing accuracy and confidence in system design differences [21].

A simulation model is characterised as a set of assumptions, in the form of mathematical or logical relationships regarding the facility of interest, usually called a system [21]. The set of assumptions form a model that is used to understand how the system behaves. Schmidt & Taylor [37] defined the state of a system as, "the collection of variables necessary to describe the status of the system at any given time". The state of a system at any point in the simulation should describe the behaviour at that instant, in some measurable way.

A system can be categorised as either continuous or discrete. The difference between these two systems is that in a continuous system, state variables change continuously over time and in a discrete system, the state variables only change at discrete points in time when an event occurs [21]. A global event queue is often used to process and manage individual events and activate components as required during the simulation of a discrete system.

Simulation models can either be deterministic, containing no random variables; where the output is "determined" once the set of inputs and their relationships have been specified, or stochastic. Simulations of real-life are mostly modelled as stochastic systems [21]. Stochastic simulations model the behaviour of some random element that cannot be precisely predicted. Stochastic simulation where the state of a system changes at discrete points in time, is called discrete-event simulation. Random variables are usually generated from a statistical distribution in discrete-event simulation to model the unpredictability of nature on event input given to the model.

2.5 The Retailer's simulation

To measure the effectiveness of dynamic size profile adjustment, a product's weekly sales simulation was needed. Dynamic size profile adjustments are initiated by the allocation process when stock arrives at the DC from factories. Thus, stock arrival needed to be incorporated into the weekly sales simulation. The system of weekly sales simulation consists of the product sold, customers that buy the product and stores where sales take place. State variables of the system change weekly and are opening stock, demand and closing stock, making the system discrete. Demand is a random element that cannot be precisely predicted. Therefore, weekly demand input is stochastic making a products weekly sales, a discrete-event simulation.

For the system of weekly sales simulation parameters of weekly demand need to be estimated from a statistical distribution of demand. Literature relating to the estimation of demand parameters are discussed in §2.5.1. Weekly demand represents the product's total demand amongst all stores (and sizes). The simulation of sales requires each unit of demand from total demand to be simulated at stores then sizes based on a sampling technique. A method of Monte Carlo sampling is discussed in §2.5.2 where store and size selection techniques are discussed.

2.5.1 Estimation of demand parameters

Several studies exist in literature where statistical methods such as Maximum-likelihood estimators (MLE) were used to estimate the parameters of different demand distributions when only sales data are available [1, 9, 27, 39]. A considerable amount of historical sales data is required to determine a statistical demand distribution that accurately represents actual customer demand [8]. In the case of limited historical sales data where a statistical distribution of demand cannot be determined, methods such as MLE are unable to estimate parameters of demand. However, the use of an underlying forecasting method in order to generate demand parameters is a possible [41].

It is essential to generate random numbers which represent demand with some given probability distribution to ensure the simulation is stochastic (to ensure the model captures a level of unpredictability associated with customer demand). Gallego *et al.* [12] analyse local and central control of a two-stage distribution system containing one warehouse and multiple retailers. Retailer's demand is stochastic and arrives following a Poisson distribution. The Poisson distribution is unique in several respects, most distinctively this distribution only requires one parameter [11]. A major assumption of the Poisson distribution is that variance is equal to the mean, which is violated if data contains excess zeros [13]. Given only the mean rate of occurrence for a certain period, the Poisson distribution generates a random variable for the event which is most likely to occur during the period of observation. The Poisson distribution is always skewed toward the right and is inhibited by the zero occurrence barrier on the left. The Poisson distribution applies when (a) the event may only be a positive integer, (b) occurrences of events are independent, (c) the average frequency of occurrence for the time period in question is known, and (d) it is possible to count how many events have occurred [21, 25, 42].

In a related study of the Retailer, Thom [40] was unable to determine a statistical distribution of weekly demand due to limited available data. A traditional quantitative technique, multiple regression, was used as an underlying forecasting method to generate weekly demand parameters. Multiple regression studies the relationship between a dependent variable and two or more independent variables. When the values of the independent variables are known, regression analysis is able to predict the mean value of the dependent variable [42].

The Poisson distribution is an acceptable method of stochastic weekly demand generation, as demand is required to be an integer and may not be negative, weekly demand in the Retailer's simulation is independent. Furthermore, the mean demand for each week is known from the regression equation and the simulation model records the number of weeks that have already been simulated.

2.5.2 Sampling technique

Monte Carlo is classed as a technique of statistical estimation. Monte Carlo simulation is related to discrete-event simulation in that it is a stochastic process [14]. Unlike discrete-event simulators which are often used to model deterministic systems, Monte Carlo simulators can be used to model non-deterministic systems where probability plays a major role [2]. Monte Carlo sampling is the procedure of selecting a point from a set so that each point in the set has a specified probability of being selected representative of each point's fitness relative to the population [22]. If f_i is the fitness of point i in the population, the probability of point i being selected is $p_i = \frac{f_i}{\sum_{i=1}^{N} f_i}$, where N is the number of individual points in the population and $\sum\limits_{i \in \mathcal{I}} p_i = 1$.

Roulette-wheel selection exists within Monte Carlo sampling and follows the analogy of a roulette game. Roulette-wheel selection is based on pseudo randomness and probabilistic weighting. The roulette wheel contains a number of compartments equal in size to the population, where each compartment is proportional to probability p_i for each point in the population [26]. A uniform random number is generated and the compartment interval corresponding to the generated random number is selected [42]. A random number is generated to imitate the randomness associated with spinning a roulette-wheel.

For the system of weekly sale simulation, total demand is apportioned first to a store level using roulette-wheel sampling, and then the store demand is apportioned to a size level, also through roulette-wheel sampling. Sampling from discrete distributions is based on the frequency interpretation of probability and the procedure should be independent (non-deterministic) [42]. Meaning, all store then size selections will occur with frequencies specified by the probabilities associated with store and size distributions and the selection of one store and size will not influence the selection of another store and size.

The probability of a given unit of demand occurring at store t is based on the historical proportion of store t's demand relative to total demand at all stores, as well as availability. A weight, w, is associated with the historical proportion of demand and a weight $(1 - w)$ with availability [40]. The value of weight, w, in the calculation of store probabilities is $w = 0.99$. Thom [40] experimented with value variation sensitivity analysis of w and found no significant effect on the total sales simulated. However, a small weight is not advised as it would artificially increases the probabilistic weighting associated with store demand by placing too much importance on availability. A value of $w = 0.99$ ensures spacial demand remains within historical geographical demand. The probability of a given unit of demand occurring in size s at store t is based on the historical proportion of size s's demand relative to total size demand at the chosen store. Availability does not influence size demand, for example a customer's shoe size does not change from a 4 to 7 if only size 7 is available.

CHAPTER 3

Methodology

Contents

This thesis aims to improve anticipated demand by dynamically adjusting size profiles as current sales data becomes available throughout the selling season. This is driven by the Retailer's allocation process in a manner that reflects the current/actual sales performance of the store, throughout the season. This chapter provides a description of dynamic size profile adjustment, and where it fits into the simulation model developed to analyse the effect of dynamic size profile adjustments on total sales for the company.

Two summer and two winter subclasses are considered in this thesis. Sales for each subclass are simulated independently of one another, following the same simulation logic. Data on orders, allocation and sales for each of the subclasses were provided by the Retailer, a description of the data available may be found in §3.1. The data is used to build simulation models for each subclass and to verify the model validity.

Weekly sales are simulated on a subclass level, meaning all styles relating to a particular subclass are handled together. The simulation model is built in Python 3.6.3 [32], a description of the simulation model is available in this chapter. The system being simulated consists of a set number of weeks, stores and sizes. A schematic representation of the simulation logic from the perspective of a retail store is presented in Figure 3.1.

The light green boxes in Figure 3.1 indicate the scope of weekly processes executed by the simulation model for all stores. Starting from the left hand side of the schematic, the simulation

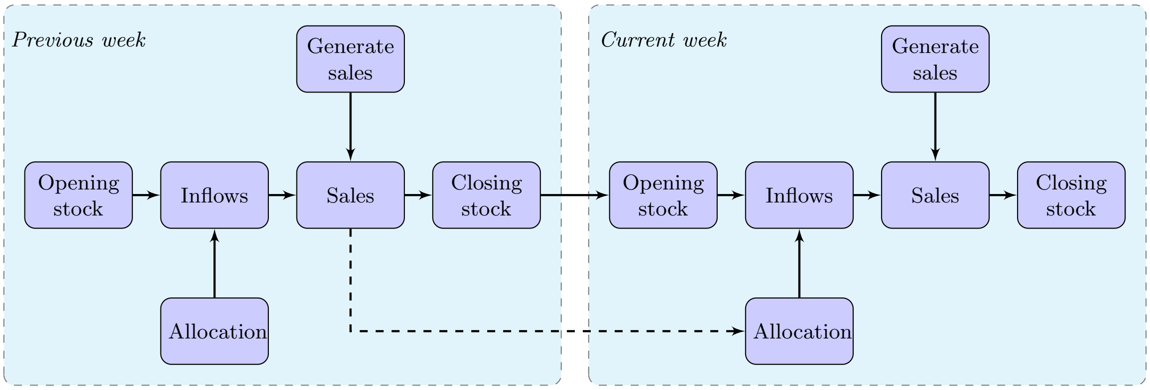

FIGURE 3.1: *A schematic representation of processes for the discrete-event simulation model for a retail store.*

model's first process is to determine `Opening stock` for every size in a store. In the first week of the simulation, `Opening stock` is initialised with a value of zero as the model assumes there is no carry over stock from the previous season. For each successive week, `Opening stock` is equivalent to `Closing stock` from the previous week.

The second process executed weekly by the simulation model are `Inflows`, which is zero for each size in a store, for all stores; unless stock is available at the DC. For any particular week, if stock arrives at the DC from factories `Inflows` are determined by the `Allocation` process, for each size at a store, for all stores. The `Allocation` process is tasked with partitioning a fixed company size-mix into smaller size-mixes for stores. The Retailer's allocation process considers the amount of stock available at the DC, anticipated demand—estimated by the Retailer months before stock arrives at the DC—and stock constraints. The main objective of this thesis is to analyse the effect of incorporating current/actual sales performance into the `Allocation` process, with the aim of improving sales. In the simulation model, decisions made during the `Allocation` process use available `Sales` information, recorded for each size at a store from the start of the simulation model until the previous week, enabling dynamic size profile adjustments. The `Allocation` process is described in more detail in §3.2.

Once `Opening stock` and `Inflows` have been calculated and updated for each size in all stores, the `Sales` process is amended depending on `Generate sales` outcome. A comprehensive illustration of the `Generate sales` process is available in §3.3, followed by a description of the processes utilised to create input for the `Generate sales` process in §3.4.

Weekly, a retail store's `Sales` process records each unit of sale that is generated for a specific size at the store. A complete collection of weekly sales information generated throughout the simulation is retained in each store's `Sales` process, enabling store specific simulated sales information to be incorporated into the `Allocation` process, indicated by the dashed line from `Sales` to `Allocation`.

At the end of each week, a store's `Closing stock` is calculated per size, indicating the amount of stock remaining in each size at the store. Throughout the season, `Closing stock` serves as the following week's `Opening stock`, per size in a store. In the last week of the season, `Closing stock` reflects the amount of unsold stock in each size at a store.

3.1 Data

Two summer products and two winter products, known as subclasses, are considered in this study. Data on the orders, allocation and sales for each of the subclasses were provided by the Retailer. Table 3.1 lists the unique ID and description for each subclass. The season is also noted as the sales characteristics differ depending on the time of year. In the final column, the range of available years data received from the Retailer is listed. This study keeps the last available year of each subclass as holdout data to verify the simulation model, so that at least three years of historical data are available when building the simulation model.

Subclass ID	Subclass description	Season	Available years
S1	Ladies fancy sandals	Summer	2010–2014
S2	Mens fancy sandals	Summer	2011–2014
W1	Teenage girls fancy slippers	Winter	2011–2014
W2	Ladies spun ploy jackets	Winter	2011–2014

TABLE 3.1: *Properties of subclass data received from the Retailer.*

Each subclass consists of a number of styles that are sent throughout the season. This study uses order and allocation data for the holdout period to finalise allocation decisions about where and how much stock to send to stores as new styles arrive in the DC. To ensure comparability across seasons for each subclass, the order, allocation and sales data needed to be cleaned.

3.1.1 Order data

The Retailer provided data specifying the `date of stock arrival` for each style in a subclass from factories to the DC, and the `quantity of stock` that arrived for each of the styles. Included in the data set are `unique style codes` for stock arriving throughout the selling season and, the year and season in which styles arrive at the DC.

In the case of duplicate style codes, a unique identifier needed to be assigned to distinguish between allocation requirements. One summer Subclass, S1, and one winter Subclass, W2, each had two duplicate style codes. These duplicates were each replaced with a unique code which allowed the allocation model to accurately identify the styles arriving on each date, no other influence on the model outcome occurs from the replacement of duplicate style codes. Apart from replacing duplicate style codes, no other data cleaning was necessary for the order data sets.

It is assumed that stock arriving at the DC is allocated to each store, for all stores planned to receive the style in the DC with no time delay in the simulation model. Inflow arrivals at stores vary, meaning no distinct pattern or rule of stock allocation amongst stores could be identified from the data. Thus, the assumption of zero lead time is made, creating unchanging weekly stock allocation events that test the effect of dynamic size profile adjustments on total sales. A potential decrease in sales may occur in the simulation model due to the Retailer's knowledge that "freshness" sells, meaning frequent stock inflows increase customer demand. However, not including the assumption of zero lead time inhibits an effective analysis of dynamic size profile adjustments as an increase in sales, could be attributed to "freshness" rather than an improvement of stock allocation. The proposed simulation model with this assumption is validated to generate output sufficiently close to the real system considered in this study.

3.1.2 Allocation data

As stated previously, each subclass has a number of styles that are sent throughout the season. Not all styles in a subclass are sent to all stores as style demand differs between stores. Allocation data for each style contains the `store numbers` which are planned to receive stock of the style and each store's relevant information, used to assist in the allocation process. The relevant information in each style allocation data set includes, for each store; `preliminary allocation` (based on anticipated demand for the store, for the style), `store grading bounds`, `expected rate of sales`, and the `size profile` (expected spread of sales as a percentage per size, for the store based on anticipated demand).

The preliminary allocation per store is calculated so that the number of units ordered from the factory is equal to total anticipated demand at all stores. The Retailer makes use of a grading system based on historic store turnover. The grading system provides upper and lower bounds, referred to as "grade minimum" and "grade maximum" for stores. These bounds ensure each store receive at least a minimum and no more than a maximum stock inflow for each style allocation. The expected rate of sales is given in number of units per week at each store and is known over time

Regardless of style, stores with similar historical sales properties are clustered together per subclass. Each cluster has an associated size profile which represents the expected spread of sales across sizes and is given as a percentage per size. During the final allocation process, size profiles enable total stock (fixed company size-mix) available at the DC to be partitioned to stores, in a way that reflects the expected spread of sales per size at each store.

Duplicate style code identifiers replaced in §3.1.1, order data sets were similarly replaced for the corresponding allocation data sets, ensuring consistency of identifiers overall data sets. Table 3.2 presents the cleaned number of unique styles for each subclass (No.styles), along with the number of styles allocated. Some styles had no allocation data and actual allocations received from the Retailer were used in the place of solutions that would have been generated by the allocation algorithm. Allocation data was not available for one style in S1, for two styles in W1 and for one style in W2. The final column in Table 3.2 indicated the remaining number of styles to be allocated per subclass for the season.

Subclass ID	No. styles	Styles allocated
S1	13	12
S2	3	3
W1	11	9
W2	12	11

Table 3.2: *Properties of subclass style data used in this study.*

It is assumed that the Retailer's actual inflow for styles without allocation data are the same as size-mix allocation solutions that would be determined for any adjustment to size profiles. Not dynamically adjusting size profiles for these styles may negatively impact potential results. However, removing styles without allocation data would decrease sales generated via a simulation model. As style information is not recorded in the Retailer's actual sales data, it is not possible to remove the data relating to these styles. Thus, the simulation model would not be sufficiently accurate in generating sales. The proposed simulation model is validated to generate sales that are sufficiently close to the real system when the Retailer's actual inflow is used for styles without allocation data.

3.1.3 Sales data

At most four years of historical sales data were provided by the Retailer for each subclass. This study makes use of historical sales data to build a simulation model that emulates historical sales, and to test validity of the model. A summary of properties associated with sales data for each subclass is available in Table 3.1. The last available year's data is kept as holdout data to verify simulation accuracy.

All available sales data was cleaned in a cohesive manner to ensure data used for model building and validation have corresponding characteristics. Holdout data sets containing unit sales recorded before the simulation model allocates the first style of the season is replaced with a value of 0 in the holdout data sets, to ensure that any sales recorded before the first allocation do not skew the interpretation of simulation model output.

Winter sales start either in the first or second week of February and summer sales start either in the last week of July or the first week of August, both seasons lasting for 26 weeks. The Retailers considers each Sunday as the last day of the week. Sales data are recorded by the Retailer every Sunday, for each size at every store in the company. A maximum of four year's historical sales data are available for each subclass considered in this study. Weekly `sales` per size are recorded for every store in the subclass, along with `opening stock`, `inflows`, and `closing stock`, as a number of units stock.

Table 3.3 presents a summary of the cleaned data sets for each subclass. Final datasets only included stores that (a) have at least one year of historical data, (b) appear in the holdout data set and, (c) receive at least one style allocation during the holdout year. In other words, new stores with no historical data, stores that closed down and stores with no planned stock allocation during the holdout year were (all) removed from the datasets.

Subclass ID	No. styles	No. stores	No. sizes
S1	13	1 279	6
S2	3	969	5
W1	11	1 273	6
W2	12	950	6

TABLE 3.3: *Properties of cleaned subclass data used in this study.*

Some datasets had incomplete sales data for one or two sizes, meaning the Retailer decided to either expand or reduce the number of available sizes during at least one of the historical seasons. Datasets were cleaned so that only sizes with complete data for all available years were included. No sizes were removed from Subclass S1, one size (size 11) had incomplete records amongst stores overall Subclass S2 historical years data and was removed. In 2014 a new size (size 9) was introduced for Subclass W1 and needed to be removed from the data. Two sizes (size 44 and 46) had to be removed from Subclass W2 as these sizes were only introduced from 2013. Subclasses S1, W1, W2 had six remaining sizes and S2 had five sizes remaining.

3.2 Allocation

This thesis aims to improve anticipated demand by dynamically adjusting size profiles as current sales data becomes available throughout the selling season. To analyse the effect of dynamic size profile adjustment, weekly sales are simulated. The simulation model records opening stock, inflows, sales and closing stock of each size at every store weekly. Inflows for each size in a store are zero, unless stock is available at the DC in which case the allocation process calculates and updates inflows for each size in every store. A schematic representation of the allocation process is presented in Figure 3.2, where the large light blue box encompasses the allocation algorithm.

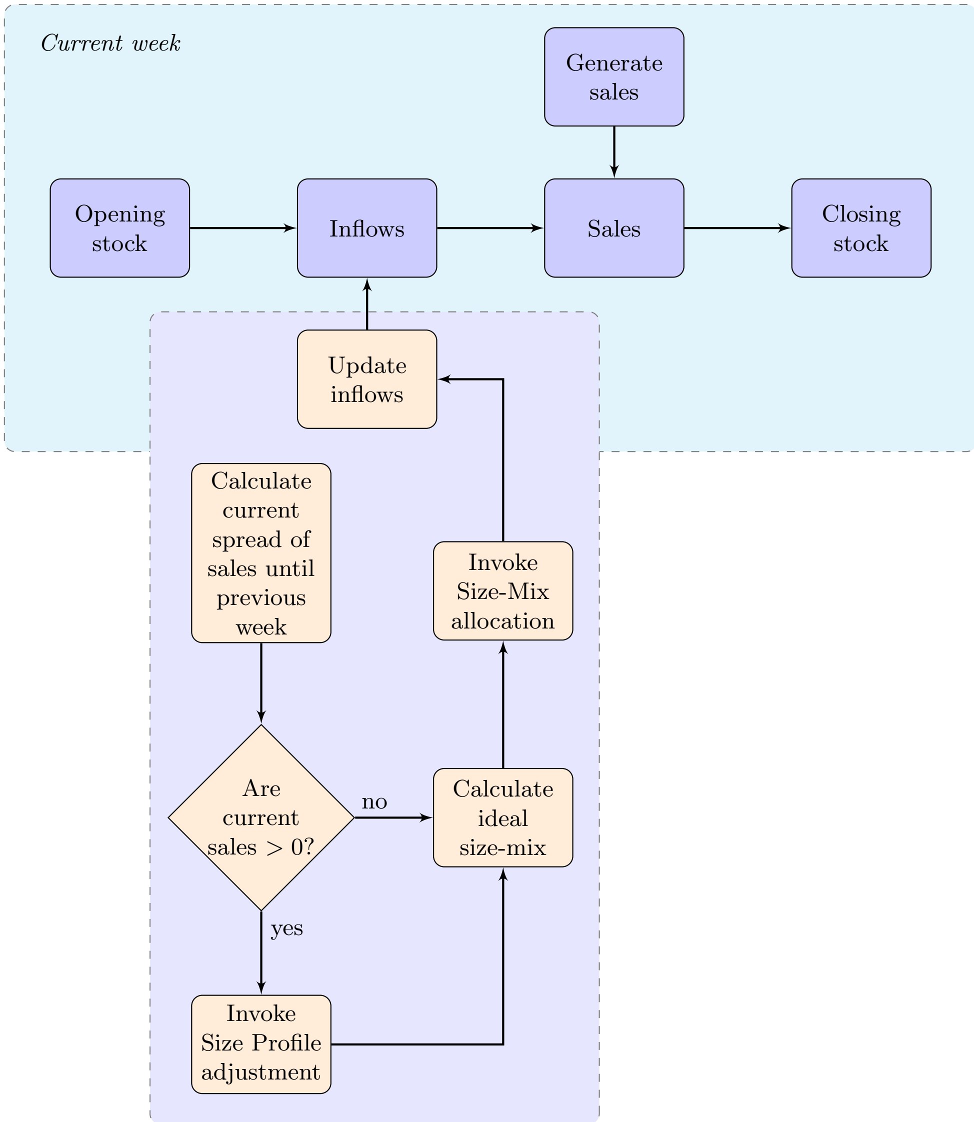

FIGURE 3.2: *A schematic representation of the allocation process/algorithm within the discrete-event simulation model.*

To enable the dynamic adjustment of a store's size profile, the allocation algorithm first calculates the current spread of sales across sizes at the store, using sales data available (until the previous week). Weekly, a retail store's sales process records each unit of sales that is generated for a specific size at the store. A complete collection of weekly sales information generated throughout the simulation is retained for each store in the sales process. At the time of allocation, this sales information is given as input to the allocation algorithm to calculate the current spread of sales at a store, until the previous week. If a store has not received inflows previously in the simulation or if no sales have been recorded to date, the current spread of sales will be zero and the size profile, as determined by the Retailer; is not adjusted/remains static. However, if sales have been recorded at a store the allocation algorithm invokes size profile adjustment, which determines a new size profile for the store. The methodology of size profile adjustment is available in §3.2.1.

Regardless of whether a store's size profile remains static or is dynamically adjusted, the allocation algorithm calculates an ideal size-mix using the store's preliminary allocation and size profile. The ideal size-mix calculated for each store assists the allocation process in partitioning a fixed company size-mix into smaller size-mixes for stores. Once each store's ideal size-mix has been calculated the allocation algorithm invokes size-mix allocation. The size-mix allocation formulation is created in Python by extracting relevant stock constraints from allocation data and inserting the calculated ideal size-mix. Inflows are finalised by solving the size-mix allocation using the CPLEX Python API [17, 18]. The size-mix allocation aims to send stock to stores as close as possible to the calculated ideal size-mix for each store, while considering the amount of stock available at the DC and stock constraints. The complete mathematical formulation of size-mix allocation are available in §3.2.2. Once a feasible solution to the size-mix allocation has been determined, inflows for each size in every store (for all stores planned to receive the style available) are updated with the calculated allocation. Thereafter, the simulation model is able to move onto the next process, which is to generate sales.

3.2.1 Size profile adjustment

This thesis aims to improve anticipated demand by dynamically adjusting size profiles as current sales data become available throughout the selling season. Size profiles are the foundation of size-mix allocation decisions as they facilitate the calculation of ideal size-mixes, together with preliminary allocations. A store's ideal size-mix is ultimately what the size-mix allocation aims to send to stores, as this reflects a store's anticipated demand on a size level.

To ensure size profiles adjust in a way that does not overcompensate for either shortages or surpluses, a weighting parameter γ, balances historic and current sales data. The weighting parameter is incorporated to secure an appropriate ratio between the size profile determined by the Retailer (historic size profile) and the current/actual spread of sales as recorded weekly from the start of the season until the current week (simulated size profile).

Define the set $\mathcal{B} = \{1, 2, ..., b, ..., B\}$ as the set of stores able to receive size profile adjustments, and $\mathcal{S} = \{1, 2, ..., s, ..., S\}$ as the set of sizes in the model. The following variables are also defined. Let

H_{bs}	be the historic size profile for size s at store b as calculated by the Retailer,
A_{bs}	be the simulated size profile for size s at store b as recorded for all weeks until the current week, and let
N_{bs}	be the adjusted size profile for size s at store b.

The size profile adjustment equation is given by

$$N_{bs} = \gamma H_{bs} + (1 - \gamma) A_{bs}, \tag{3.1}$$

where γ, is associated with the historic size profile and $1 - \gamma$, is associated with the simulated size profile.

The value of γ influences the magnitude of movement from the historic size profile. A suitable value of γ that effectively increases total sales without overcompensating for either shortages or surpluses needs to be determined. Sensitivity analysis must be performed for each subclass to determine an appropriate value of γ.

A small value of γ (*i.e.* $\gamma < 0.5$), places less weight on the historic size profile indicating the adjusted size profile will reflect a higher degree of the more recent size profiles. In contrast, a larger value of γ (*i.e.* $\gamma \geq 0.5$) places less weight on the more recent size profile resulting in an adjusted size profile that is more similar to the historic size profile.

3.2.2 Size-mix allocation

A mixed-integer programming formulation of size-mix allocation calculates inflows for each size in a store, for all stores planned to receive the style available at the DC. The objective is to maximise the company's (all sizes and stores) expected number of sales for the style. The effect on total sales as a result of inflows calculated using the size-mix allocation presented in this section is not significantly different from total sales recorded by the Retailer [40]. As the allocation formulation used by the Retailer is unknown, the size-mix allocation presented here is sufficient in calculating inflows.

The size-mix allocation considers stock available at the DC, anticipated demand and stock constraints specified by the Retailer. Assumptions regarding the data received from the Retailer had to be made.

1. Preliminary allocation for each store is made based on anticipated demand by the Retailer, is a good approximation of expected sales. Demand is considered to be deterministic and known from forecasts.

2. Size profiles as determined by the Retailer, based on anticipated demand are good approximations to the expected spread of sales across sizes. Size profiles will dynamically adjust as the season progresses and are used to calculate ideal size-mixes.

3. The rate of sales at each store, and size within a store, is known over time and approximately equal to the rate of sales provided by the Retailer. Store and size rate of sales are expressed as the expected number of units that will be sold per week.

4. Allocations for each style are done once in the season as a whole, meaning no stock of the style has been allocated previously to stores.

The following parameters are used. Let

d_b be the preliminary allocation at store b,
d_{bs} be the ideal size-mix for size s at store b,
b_s be the total number of units of size s that are available at the DC,
r_b be the expected number of units that will be sold per week at store b, provided by the Retailer,
r_{bs} be the expected number of units of size s that will be sold per week at store b, provided by the Retailer,
g_b be the minimum number of units that may be sent to store b, according to the Retailer's grade minimum requirements,
h_b be the maximum number of units that may be sent to store b, according to the Retailer's grade maximum requirements,
m_b be the maximum deviation from d_b specified by the Retailer, measured in number of weeks' stock, and let
m_{bs} be the maximum deviation from d_{bs} specified by Retailer, measured in number of weeks' stock.

Define the following variables. Let

x_{bs} be the number of units of size s that are sent to store b, and let
y_{bs} be the expected number of sales of size s at store b.

The mathematical formulation of the model is given by

$$\text{maximise } z = \sum_{b \in \mathcal{B}} \sum_{s \in \mathcal{S}} y_{bs} \tag{3.2}$$

subject to

$$d_{bs} \geq y_{bs}, \qquad b \in \mathcal{B}, s \in \mathcal{S} \tag{3.3}$$

$$x_{bs} \geq y_{bs}, \qquad b \in \mathcal{B}, s \in \mathcal{S} \tag{3.4}$$

$$\sum_{b \in \mathcal{B}} x_{bs} = b_s, \qquad s \in \mathcal{S} \tag{3.5}$$

$$\sum_{s \in \mathcal{S}} \sum_{b \in \mathcal{B}} x_{bs} = \sum_{s \in \mathcal{S}} b_s \tag{3.6}$$

$$\max(g_b, d_b - r_b m_b) \leq \sum_{s \in \mathcal{S}} x_{bs} \leq \min(h_b, d_b + r_b m_b), \qquad b \in \mathcal{B} \tag{3.7}$$

$$d_{bs} - r_{bs} m_{bs} \leq x_{bs} \leq d_{bs} + r_{bs} m_{bs}, \qquad b \in \mathcal{B}, s \in \mathcal{S} \tag{3.8}$$

$$x_{bs}, y_{bs} \in \mathbb{Z}^+, \qquad b \in \mathcal{B}, s \in \mathcal{S}, \tag{3.9}$$

where $\mathbb{Z}^+$ is the set on non-negative integers.

The objective function (3.2) maximises the expected number of sales for all stores and sizes in the style being allocated. The expected number of sales are calculated in constraint sets (3.3) and (3.4), where the expected number of sales are constrained to be smaller than or equal to the ideal size-mix, and the number of units sent to each size at every store. A store's ideal size-mix reflects the expected sales of sizes at the store, determined using anticipated demand which includes the store's preliminary allocation and size profile. Size profiles can either be static, as determined by the Retailer or dynamic, determined by size profile adjustment.

Constraint set (3.5) ensures the total number of units sent in each size is equal to the total amount of stock available at the DC in a size (fixed company size-mix). Similarly, constraint (3.6) ensures that the total number of units sent for a style equates to the total amount of stock available at the DC for that style, preventing misallocation of unavailable stock and ensuring all units of available stock are sent to stores.

Constraint sets (3.7) and (3.8) ensure adequate stock is sent on a store and size level according to stock constraints, in number of weeks' shortages and surpluses; specified by the Retailer. The Retailer's grading system is also included in constraint set (3.7).

3.3 Generate sales

The Retailer's sales are random, mainly due to the direct influence of customer demand. To model the unpredictability of sales, a random variable must be generated from a statistical distribution that reflects the Retailer's weekly demand. A schematic of the process/algorithm to generate sales is presented in Figure 3.3, where the large light blue box encompasses the generate sales process, executed weekly throughout the simulation.

Once the company's opening stock and inflows (each size, at every store) have been calculated and updated, the simulation model generates sales which emulate historical sales. Given only the average rate of occurrence for a certain period of observation, the Poisson distribution generates a random variable most likely to occur during that period. Weekly, a regression forecasted value reflecting the Retailer's mean demand ($\hat{Y}_k$) for the period of observation (current week) is given as the input parameter for a Poisson random variable (D_k). Forecasted demand is derived from historical sales, as indicated by the flow processes encompassed by the light red box in Figure 3.3. The processes within this red box are executed once and the resulting parameter ($\hat{Y}_k$), given weekly as input to generate sales, are calculated before the simulation model is run. A description of the processes to create the Retailer's mean demand parameter are available in §3.4.

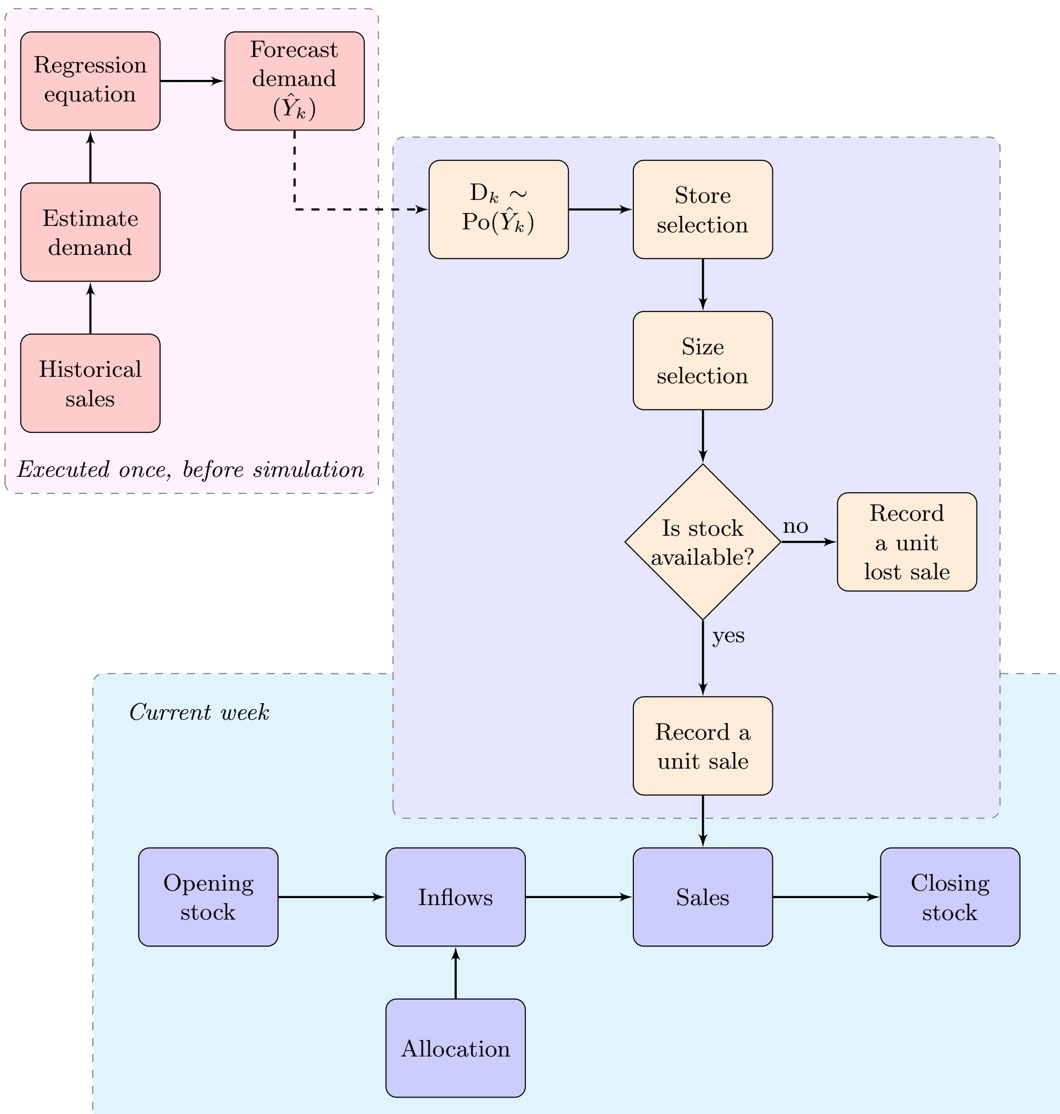

FIGURE 3.3: *A schematic representation of the generate sales process/algorithm within the discrete-event simulation model.*

Within the large light blue box in Figure 3.3, once the random weekly demand variable, D_k, has been generated using the Poisson distribution (first yellow block), total demand for the week is segmented into individual units corresponding with stores and sizes, emulating historical sales. Roulette-wheel selection, also called stochastic sampling with replacement follows the analogy of a roulette game and is based on pseudo-randomness, and probabilistic weighting. Each segmented unit is simulated first on a store level using roulette-wheel selection, followed by a size selection within the store again using roulette-wheel selection. If stock is available in the selected size at the store, a unit of sale is recorded by the simulation model in the selected store's sales process for the selected size, otherwise a unit of lost sale is recorded.

Each store in the company has an associated probability of being selected, calculated at the start of each week and derived from the historical contribution of the store's demand to the total demand of all stores, as well as the amount of opening stock and inflows at the store for the week. The ratio of historical demand to availability is 0.99 to 0.01, enabling dynamic adjustment of size profiles (and the resulting inflows) to have a 0.01 influence on the calculation of store selection probability. The probability of size selection within a store is derived from the the historical contribution of a size's demand to the total demand of all sizes within a store.

3.4 Generate sales input

The flow of processes to create input parameters for generating sales in the simulation model are presented in this section. Demand data need to be calculated and estimated as there is no reasonable way to acquire actual demand data. The following assumptions had to be made regarding data received from the Retailer for the purpose of creating input parameters to generate sales.

1. Future demand (parameters of weekly demand) can be derived from historical sales data, based on a regression equation built using estimated demand.

2. Historical sales are used to estimate demand. However, in the case of a stockout, demand is assumed to decrease linearly to zero during the next three weeks. Demand has to be estimated in the case of a stockout and cannot simply to assumed to be equal to sales. A linear decrease is assumed in the absence of any additional information or data to estimate demand.

Due to limited historical sales data available from the Retailer, and based on Assumption 1 and 2, parameters of weekly demand are derived from historical sales data. Figure 3.3 indicates the flow of input creation processes, starting with historical sales which result in input parameters of weekly demand, used to generate sales for the simulation model (dashed line indicates input). The calculation used to estimate demand parameters, followed by the methodology and assumptions associated with creating regression equations are described in the next sections.

3.4.1 Estimate demand

Historical sales data for the four subclasses considered contain at most four years' weekly recording of opening stock, inflows, sales and closing stock for each size at every store. Based on Assumption 2, a calculation which adjusts for stockouts is used to estimate demand from historical sales data.

Define the set $\mathcal{K} = \{1, 2, \ldots, k, \ldots, K\}$ as the set of weeks in a season, The following variables are also defined. Let

f_{sbk} be the number of weeks of stockouts remaining for size s, at store b, in week k

(including week k),

n_{sb} be the number of units of demand for the next few weeks in the case of a stockout for size s, at store b, and

d_{sbk} be the estimated demand for size s, at store b, in week k.

When stock is not available, a non-negative demand is estimated for the number of weeks of stockout left (including week k) or until either new stock arrives or the season ends (f_{sbk}). The number of units of demand for the next few weeks (n_{sb}), is calculated by multiplying the number of weeks of stockout left (f_{sbk}) with average sales (of the previous three weeks) for the specific size at a store where stock is not available. The ceiling of this calculation is taken, ensuring at least one unit of demand is estimated when equation (3.10) is used. Each subsequent week where stock remains unavailable, for the specific size at the store; the calculated number of units demanded is updated by subtracting estimated demand during the previous week (d_{sbk-1}) from the the initial calculated (total) number of units demanded (n_{sb}).

$$d_{sbk} = \min\left\{\left\lceil \frac{n_{sb} \times f_{sbk}}{(f_{sbk}(f_{sbk}+1))/2} \right\rceil, n_{sb}\right\} \tag{3.10}$$

Equation (3.10) ensures estimated demand gradually dies out from an average of the previous three weeks sales to zero. Estimated demand is at most equal to the calculated number of units demanded in a week (n_{sb}). Demand is required to be an integer, therefore rounding of the calculation is necessary. It is estimated that three weeks are necessary for demand calculations based on the presumption that stock will be replenished at least every three weeks, meaning demand is estimated for a maximum of three weeks.

3.4.2 Multiple linear regression

The multiple linear regression model is represented mathematically as an algebraic relationship between a response (dependent) variable and two or more predictor (independent) variables [29]. To model the unpredictability of sales, parameters of weekly demand need to be estimated from a statistical distribution of the Retailer's demand. At most four years of demand data was available (as calculated in §3.4.1 as estimated demand) and each week followed a different distribution. In the case of limited demand data, multiple regression is an appropriate method of forecasting weekly demand parameters through the use of a regression equation. Each subclass' resulting regression equation forecasts weekly demand for the holdout period, using its estimated demand as the dependent variable and observed patterns from the time-series data of estimated demand, as explanatory variables. The general formulation of the regression equation is as follows.

Let Y represent the value of the dependent variable, $\hat{Y}$ the predicted value of the dependent variable and X_i the value of the i^{th} independent variable. Then the population multiple regression equation is given by

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \ldots + \beta_i X_i + \epsilon,$$

where β_0 is the intercept, β_i are the slopes associated with X_i for all i and $\epsilon = Y - \hat{Y}$ is the population error term. The error term should follow a normal distribution with mean 0.

At most three years of historical sales data are available, therefore values for the slopes of β_i are estimated from sample data. The estimates for β_i are represented by $\hat{\beta}_i$ for all i. Then the prediction for $\hat{Y}$ is given by

$$\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \hat{\beta}_2 X_2 + \ldots + \hat{\beta}_i X_i.$$

To find values for estimates $\hat{\beta}_i$, the method of least squares is applied. Let $\mathcal{J} = \{1, 2, \ldots, j, \ldots, J\}$ be the set of observations. Then the values for $\hat{\beta}_i$ may be estimated by minimising the sum of the squared errors for all observation in set $\mathcal{J}$, in other words, by minimising

$$\begin{aligned}\sum_{j \in \mathcal{J}} \epsilon_j^2 &= \sum_{j \in \mathcal{J}} (Y_j - \hat{Y}_j)^2 \\ &= \sum_{j \in \mathcal{J}} (Y_j - \hat{\beta}_0 - \hat{\beta}_1 X_{1j} - \hat{\beta}_2 X_{2j} - \ldots - \hat{\beta}_i X_{ij})^2,\end{aligned}$$

where ϵ_j is the error of the j^{th} observation, Y_j is the j^{th} dependent variable, $\hat{Y}_j$ the j^{th} predicted value and X_{ij} the value of the i^{th} independent variable for the j^{th} observation.

Accuracy of the regression model can be determined by investigating the coefficient of determination, R^2. This value indicates how well the regression line fits the data by measuring the variation in the dependent variable explained by the independent variables. An R^2 value close to 1 indicates a very good fit. The value of R^2 can become deceiving in multiple regression as additional independent variables artificially inflate the R^2 value, without the model necessarily becoming more accurate. Therefore adjusted R^2, which adjusts the statistic to account for the number of independent variables, is inspected in conjunction with R^2 when analysing the regression model.

A further test of model accuracy is the joint explanatory power of independent variables, which indicate the overall significance of the regression model and is tested by means of the F hypothesis test, given by

$$\begin{aligned}&H_0 : \beta_1 = \beta_2 = \ldots = \beta_i = 0, \text{ and} \\ &H_a : \text{at least one } \beta_i \neq 0.\end{aligned}$$

The F-statistic and corresponding p-value are calculated in SAS 9.4 [36]. Rejecting the null hypothesis in favour of the alternative means that at least one of $\beta_1, \beta_2, \ldots, \beta_i$ is not equal to zero. In this case it is concluded that at least one of the independent variables, $X_1, X_2, \ldots, X_i$, is linearly related to the dependent variable, Y. Meaning the regression equation provides a better fit of the data than a model containing no independent variables.

Once the overall model is proven to be a good fit for the data, each coefficient (describing the mathematical relationship between each independent variable and the dependent variable), is inspected and hypothesis testing is conducted to determine the suitability of each independent variable, by considering the corresponding p-value. For each independent variable i, H_0 is the null hypothesis and H_a the alternative hypothesis [42]. Therefore,

$$\begin{aligned}&H_0 : \beta_i = 0, \text{ and} \\ &H_a : \beta_i \neq 0.\end{aligned}$$

If β_i is 0, it means that the i^{th} independent variable has no influence on the dependent variable when used in conjunction with the other variables. Therefore, if H_0 is rejected in favour of H_a, it means that the i^{th} independent variable has a significant explanatory effect on the dependent variable. Variables included in a regression equation should have significant test statistics. The t-statistic for each independent variable i is given by

$$t = \frac{\hat{\beta}_i}{\text{StdErr}(\hat{\beta}_i)}$$

where $\text{StdErr}(\hat{\beta}_i)$ is the standard error of $\hat{\beta}_i$ (measured the amount of uncertainty present in the estimate of β_i). The null hypothesis H_0 is rejected if $|t| > t_{(\alpha/2,n-k-1)}$, where α is the significance level, n the number of observations and k the number of independent variables.

For the multiple regression model to be valid a handful of assumptions (regarding multiple linear regression) need to be satisfied. The assumptions are listed.

1. The regression model is linear in the parameters.

2. There is no heteroscedasticity. The error terms are constant as the value of the independent variable increase.

3. The error terms of the regression are normally distributed with a mean value of zero.

4. There is no autocorrelation. There is no positive or negative correlation between any two residuals corresponding to different observations.

5. There is no multicollinearity in the independent variables. Meaning there is no linear relationship between two different independent variables.

These assumptions are tested for regression models developed for each subclass in Chapter 4 during White-box verification and validation, discussed in §4.3.

CHAPTER 4

Verification and validation

Contents

A model that simulates the Retailer's weekly sales, emulating historical sales was built to test the effect of dynamic size profile adjustments on total sales. The simulation logic records a unit of sale if the following conditions are satisfied; (i) a unit of demand is generated and, (ii) there is stock available for the demanded unit.

Verification and validation of a simulation model are performed to establish confidence in the simulation and respective results [34, 35]. Verification, tests that all specified real-life system requirements are met, and validation is useful in establishing that the model output satisfies the true needs and expectations of the study. There are four main processes involved with the verification and validation of a simulation model, namely conceptual model validation, data validation, White-box verification and validation, and Black-box validation.

4.1 Conceptual model validation

This stage of simulation validation checks the scope and relevant detail to meet the proposed model objectives are sufficient, and that assumptions are correct [34]. Analysing data available from the Retailer and consulting expert knowledge of the real world system, ensured the conceptual model possesses sufficient detail to meet the study's objectives while representing reality satisfactorily, and that assumptions are realistic.

A discrete-event simulation model is built to analyse the effect of dynamic size profile adjustments on total sales. The model simulates sales weekly on a subclass level, meaning all styles relating to a particular subclass are handled together, for a set number of predefined weeks, stores and sizes. Aspects of the Retailer's real system considered in the simulation model include opening stock, inflows, demand and closing stock, which are stated in Chapter 3 where a schematic representation of the model is available (Figure 3.1). Weekly the simulation model records sales for a specific size at a specific store if the following conditions are satisfied: (i) a unit of demand is generated at the specific store, in the specific size, and (ii) there is stock available for the particular unit demanded. If stock is not available, the model records a unit of lost sale for the particular unit demanded.

Aspects of the Retailer's real system that are excluded from the simulation model are firstly, costs associated with the real system (*i.e.* transportation cost, holding costs, etc.) secondly, lead time associated with stock arriving at stores from the DC (it is assumed that stock arrives at stores on the same day as stock in the DC arrives from factories) and lastly, left over stock from the previous season is excluded from the simulation model (opening stock is initialised with a value of 0 at the start of each simulation run).

4.2 Data validation

Sufficient data on the real world system must be available to build the conceptual model based on mathematical and logical relationships that represent an acceptable replication of the real system [35]. Real world order, allocation and sales data provided by the Retailer were used in building the simulation model and in all experiments. The Retailer provided reliable, appropriate and a sufficient quantity of data for the purpose of this study. The data was cleaned to remove inconsistencies and mitigate potential concern, as described in §3.1. The cleaned data was not significantly different from raw data received from the Retailer.

To build the conceptual model, historical sales data containing opening stock, inflows, demand and closing stock are needed. The simulation logic is modelled using order data, specifying when stock arrives at the DC, what style has arrived and the quantity of stock that is available for allocation to stores. Allocation data sets contain different parameter values used for the calculation of inflows, during the simulation of a season. To simulate stochastic sales in a way that accurately reflects the Retailer's weekly demand, a random variable generated via a statistical distribution, reflective of the Retailer's weekly demand, was needed.

Data transformations to estimate demand were performed in the best available approaches in order to represent reality as closely as possible. The use of available data in the simulation model resulted in sufficiently accurate output of the model's behaviour for the intended purpose of this study.

4.3 White-box verification and validation

Each element in the simulation model is verified to ensure the model is true to the conceptual model and validated to ensure each element represents the corresponding real world system with sufficient accuracy.

The simulation model flow was checked continuously during model building to ensure the correct operations were performed instantaneously, as specified for each new element added. This entailed checking the code to ensure the correct data was imported by the model and simulation logic remained true to the conceptual model. Stock keeping checks of the model were performed by tracing the progress of stores throughout the simulation model, stepping though each process of the model and recording a store's opening stock, inflows, sales and closing stock which validated that each process in the simulation model have been calculated and recorded correctly, and as expected.

The method of allocation used by the Retailer is not explicitly known and inflows are determined via a mixed integer programming problem, validated in a previous study to generate results that are not statistically different from the Retailer's recorded results [40]. The dynamic adjustment of size profiles have been validated to adjust only stores where requirements of adjustment are met (only stores where sales for the season have been recorded to date) and that the adjusted size profile remains as a percentage per size within a store, equalling 100%.

Experimental testing of dynamic size profile adjustments validated that the model reacted as expected to changes in demand. In one experiment, disproportionally small demands were artificially generated for the largest size at twenty randomly selected stores and the model responded as expected: due to less demand, less sales were recorded and at the time of allocation these store's size profiles adjusted accordingly. As recorded sales for the tested size were lower than historically expected (specified by the Retailer's calculated size profile) at these stores, their size profiles were decreased by the adjustment algorithm to reflect the current sales as recorded by the simulation model. The adjusted size profile resulted in a lower unit allocation for the tested size (large size) at the stores included in the experiment, ultimately reducing the amount of left over stock recorded by the simulation model with dynamic adjustment, in comparison to recorded sales generated by keeping size profiles static in the simulation. The opposite is true for an experiment where the inverse was tested by artificially increasing demand. This resulted in an early stock out and size profiles adjusted dynamically to account for the increased demand by increasing the percentage of size profile associated with the large size in the tested stores.

An important aspect of the simulation model was to determine input parameters of the Retailer's weekly demand to simulate sales that emulate historical sales. As there is no reasonable way to acquire actual demand data, demand is estimated from historical sales data, available from the Retailer; and increased to account for lost sales. The parameters used in the simulation model were generated by an underlying regression forecasting model, as described in §3.4. As part of the simulation model verification and validation, the following section presents the regression forecasting model, for each of the subclasses considered in this study. Each of the regression equations are shown to be a good fit for the data and are validated by testing that the regression assumptions hold.

4.3.1 Subclass S1 regression validation

The following variables are defined for inclusion in the regression model for summer Subclasses S1 and S2, based on the pattern of estimated demand which confirms seasonality in the subclasses. Let $\mathcal{K} = \{1, 2, ..., k, ..., K\}$ be the set of weeks in a season and let

Y_k be the total weekly estimated demand,
L_k be the total unit inflow in week k as planned by the Retailer,
W_k be week k's week number in the year,

$$E_k = \begin{cases} 1 & \text{if the last day of week } k \text{ is after the } 29^{\text{th}} \text{ or before the } 11^{\text{th}} \text{ of a month,} \\ & \text{excluding the end of December and the beginning of January,} \\ 0 & \text{otherwise, and let} \end{cases}$$

$$C_k = \begin{cases} 1 & \text{if the last day of week } k \text{ falls in the interval from the } 17^{\text{th}} \text{ to the } 30^{\text{th}} \text{ of December,} \\ 0 & \text{otherwise.} \end{cases}$$

Weekly estimated demand, Y_k is the dependent variable and $\hat{Y}_k$ is the forecasted demand for week k, which is the resulting parameter used to generate sales in the simulation model. Explanatory variable, L_k enables the model to account for increased anticipated demand, in the form of inflows as planned by the Retailer who ordered stock from factories months in advance in preparation of the increased anticipated demand, in week k. To remove positive autocorrelation present in initial experiments, a one week lag of estimated demand, Y_{k-1} and total inflows, L_{k-1} are included. Including lagged demand (Y_{k-1}), resulted in heteroscedasticity, which is removed by using $\sqrt{Y_{k-1}}$, and $\sqrt{\hat{Y}_k}$ in the place of Y_{k-1} and $\hat{Y}_k$. Indicator variables, E_k and C_k represent patterns of increased demand at the end of the month (due to the Retailer's customers receiving salaries and wages, usually at the end of the month) and around Christmas, respectively. To handle further seasonality in the data, variable W_k takes on a value representative of the week number in the year.

Having established the variables for inclusion in the model, parameters $\hat{\beta}_1, \hat{\beta}_2, \ldots, \hat{\beta}_6$ were estimated and the final regression equation to forecast weekly demand, $\sqrt{\hat{Y}_k}$ for S1 is given by

$$\sqrt{\hat{Y}_k} = 0.73\sqrt{Y_{k-1}} + 0.00057L_k + 0.00041L_{k-1} + 0.21W_k + 9.53E_k + 17.19C_k. \tag{4.1}$$

According to the coefficient of determination, $R^2 = 0.9805$, meaning 98.05% of the variation in weekly estimated demand can be explained by the independent variables in equation (4.1). The presence of multiple independent variables causes R^2 to artificially inflate, and it is valuable to note the adjusted coefficient of determination, adjusted $R^2 = 0.9793$, which considers the effect of multiple independent variables. The adjusted R^2 value is still large and thus the model is considered a good fit for the data.

The joint explanatory power of independent variables is tested using the F-statistic and corresponding p-value, obtained via SAS 9.4 [36]. At a significance level of $\alpha = 0.05$, $p = 0.0001 < 0.05$, the null hypothesis ($H_0 : \beta_1 = \beta_2 = \ldots = \beta_k = 0$) is rejected in favour of the alternative hypothesis ($H_a :$ at least one $\beta_i \neq 0$), at least one of the independent variables is linearly related to estimated demand. Considering adjusted R^2 and the F-statistic, the model is a good fit for the data.

The signs of all coefficients are positive, as the value of any independent variable increases, the average estimated demand (dependent variable) also tends to increase. The value of coefficients signifies the change in estimated demand given a one-unit shift in an independent variable, holding all other variables in the model constant. For example, the regression coefficient associated with the last week's estimated demand ($\sqrt{Y_{k-1}}$) is 0.73; holding all other variables constant, estimated demand increases by 0.73 units for each additional unit of the previous weeks' estimated demand.

At a significant level of 0.05, Table 4.1 indicates each independent variable is statistically significant, meaning that changes in the independent variables are associated with a change in estimated demand (the dependent variable) at the population level (each independent variable has a significant explanatory effect on estimated demand).

It is concluded that the model as a whole is significant in explaining demand and the coefficients are as expected given the pattern of seasonality, the models validity must be determined by testing that the assumptions, as listed in §3.4.2 of multiple linear regression are satisfied.

Variable	t value	p value
$\sqrt{Y_{k-1}}$	24.06	$< .0001$
L_k	3.97	0.0001
L_{k-1}	2.80	0.0062
W_k	3.33	0.0012
E_k	4.37	$< .0001$
C_k	4.40	$< .0001$

TABLE 4.1: *Subclass S1 parameter estimates for regression equation (4.1), t-values and p-values.*

The coefficients in regression equation (4.1) are all constants, therefore the model is linear in parameters and Assumption 1 is satisfied.

The Breush-Pagan test is conducted to formally test for homoscedasticity in equation (4.1). For this test, the null hypothesis of homoscedasticity is tested against the alternative hypothesis of heteroscedasticity. Results of the p-value obtained from SAS 9.4 [36], is given by 0.06. Therefore, at a significance level of $\alpha = 0.05$, the null hypothesis is not rejected and it is assumed that error terms are homoscedastic, concluding that Assumption 2 is satisfied.

Graphical methods, such as the normal quantile-quantile plot (Q-Q plot) is a useful tool in checking normality for independent observations, however, graphical plots do not provide conclusive evidence that the assumption of normality holds. Therefore, four formal tests of normality, namely Shapiro-Wilk, Kolmogorov-Smirnov, Cramer-von Mises and Anderson-Darling, are performed before concluding the validity of Assumption 3. Results of the four tests of normality are obtained via SAS 9.4 [36] and presented in Table 4.2. The null hypothesis for each test states that residuals follow a normal distribution, against the alternative hypothesis of non-normality in the residuals. At a significance level of $\alpha = 0.05$, the p-value of each test is larger than the level of significance. Therefore, the null hypothesis is not rejected for each test and it is assumed that residuals are normally distributed, finalising that Assumption 3 is satisfied.

Test	Statistic	p value
Shapiro-Wilk	0.99	0.48
Kolmogorov-Smirnov	0.07	> 0.15
Cramer-von Mises	0.07	0.25
Anderson-Darling	0.46	> 0.25

TABLE 4.2: *Statistical test for normality of Subclass S1.*

A further assumption to check is autocorrelation in residuals. Autocorrelation occurs when residuals are not independent from each other. The Runs test, also known as the Geary test was performed for equation (4.1) to test for autocorrelation in the residuals. The null hypothesis of this test is that no autocorrelation is present in the data and the alternative hypothesis states there is autocorrelation. The resulting p-value of the Runs test is 0.1, the null hypothesis is not rejected at a significance level of $\alpha = 0.05$. It is assumed that no autocorrelation is present in the regression model and Assumption 4 is satisfied.

The final assumption of multiple regression states that no multicollinearity is present in the independent variables. Meaning there is no linear relationship between two different independent variables. The Pearson correlation coefficient indicates the presence of multicollinearity when the relationship between any two independent variables are either above 0.8 or below -0.8. Table 4.3 confirms that no two independent variables are highly correlated, the presence of

	$\sqrt{Y_{k-1}}$	E_k	W_k	C_k	L_k	L_{k-1}
$\sqrt{Y_{k-1}}$	1	-0.171	0.198	0.355	-0.041	0.228
E_k	-0.171	1	0.188	-0.201	0.232	0.028
W_k	0.198	0.188	1	0.3	0.222	0.277
C_k	0.355	-0.201	0.3	1	-0.028	0.011
L_k	-0.041	0.232	0.222	-0.028	1	0.085
L_{k-1}	0.228	0.028	0.277	0.011	0.085	1

TABLE 4.3: *Pearson correlation coefficients of regression equation (4.1).*

multicollinearity is not a concern for the independent variables in equation (4.1), therefore Assumption 5 is satisfied.

Regression equation (4.1) is an acceptable model to forecast weekly demand for Subclass S1 and all assumptions of multiple regression are satisfied. A graphical depiction of the fit and forecast accuracy is presented in Figure 4.1.

The Retailer's total actual demand in 2014 (the holdout period), as estimated from historical sales data is equivalent to 90 855 units. Regression equation (4.1) overestimates total demand by 2.17%, predicting a total of 92 823 units for 2014. Figure 4.1 presents a graphical display of the fit (red, dashed) and forecast (green, dashed) accuracy of regression (4.1), compared to the actual demand (blue, solid), as estimated from historical sales data; which indicate the slight over prediction is not too dramatic and regression (4.1) is concluded to be a good prediction of demand for the holdout period.

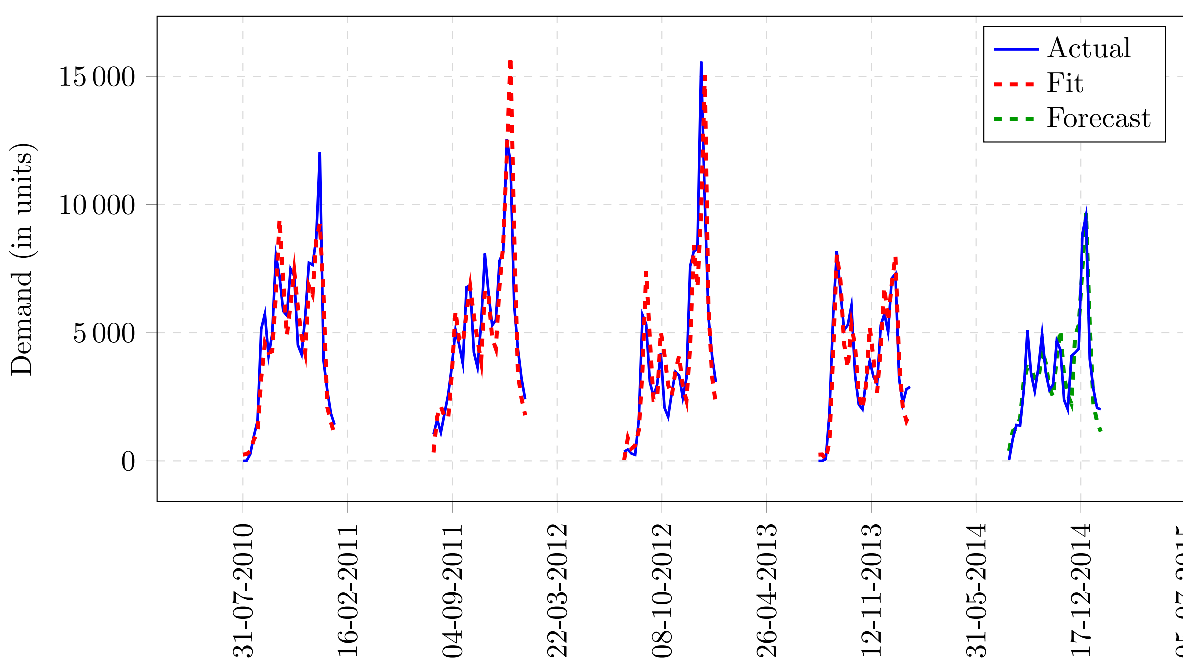

FIGURE 4.1: *Graphical display of Subclass S1 fit and forecast accuracy of regression (4.1) for the years 2010–2014.*

4.3.2 Subclass S2 regression validation

The variables defined for inclusion in Subclass S1's regression equation (4.1), when applied to Subclass S2 is given by

$$\sqrt{\hat{Y}_k} = 0.86\sqrt{Y_{k-1}} + 0.00072L_k + 0.00072L_{k-1} + 0.026W_k + 4.06E_k + 8.32C_k. \tag{4.2}$$

The signs and magnitude of the coefficients in regression equation (4.2) are as expected, following the same pattern and rationale as Subclass S1.

The R^2 and adjusted R^2 indicate a good fit, with the value of $R^2 = 0.9866$ and the value of adjusted $R^2 = 0.9718$. The joint explanatory power of the variables are highly significant as the p-value of the F-test—computed by SAS 9.4 [36]—is smaller than 0.0001.

Variable	t-test	p-value
$\sqrt{Y_{k-1}}$	33.74	$< .0001$
L_k	5.06	$< .0001$
L_{k-1}	5.26	$< .0001$
W_k	1.15	0.25
E_k	4.38	$< .0001$
C_k	5.02	$< .0001$

TABLE 4.4: *Parameter estimates for regression equation (4.2), t-test and p-value.*

The statistical significance of each independent variable is tested using the t-test and corresponding p-values as presented in Table 4.4. All but one variable are statistically significant, $p < 0.05$. Table 4.4 indicates variable W_k, week k's week number in the year, as not a significant explanatory variable of estimated demand for Subclass S2. This variable was statistically significant in the regression equation (4.1) of Subclass S1, where more historical data was available. Thus, the variable W_k shall remain in regression (4.2) for Subclass S2 where less historical data is available. The joint explanatory power of independent variables in regression equation (4.2), are significant.

The sample data from Subclass S2 provides sufficient evidence to conclude the model as a whole is significant. The validity of the regression model must be determined by testing that the assumptions, as listed in §3.4.2 of multiple linear regression hold. Assumption 1 holds as all coefficients are constant and the regression equation is linear.

Breusch-Pagan is performed and the values reported on are obtained via SAS [36]. The Breusch-Pagan null hypothesis of homoscedasticity is tested against the alternative hypothesis of heteroscedaticity. The resulting p-value is 0.0776, therefore at a significance level of $\alpha = 0.05$, the null hypothesis is not rejected and homoscedasticity may be assumed, confirming Assumption 2 holds.

Test	Statistic	p value
Shapiro-Wilk	0.98	0.29
Kolmogorov-Smirnov	0.072	> 0.15
Cramer-von Mises	0.039	> 0.25
Anderson-Darling	0.27	> 0.25

TABLE 4.5: *Statistical test for normality of Subclass S2.*

Four statistical tests of normality, reported on in Table 4.5, test whether residuals are normally distributed. At a significance level of $\alpha = 0.05$, each test of normality is greater than the level of significance ($p > 0.05$) and the null hypothesis is not rejected, concluding Assumption 3 holds.

Regression (4.2) has no intercept and the lagged dependent variable is used as an explanatory variable, therefore the assumptions of Durbin-Watson are violated and this common test of autocorrelation cannot be used. To test for autocorrelation in this equation, the Runs test is performed. Runs test is used to determine whether the residuals are from a random process, H_0: the sequence was produced in a random manner and H_a: the sequence was not produced in a random manner. The p-value of Runs test is 0.18, therefore the null hypothesis is not rejected at a significance level of $\alpha = 0.05$ and randomness in the residuals may be assumed, Assumption 4 holds.

	$\sqrt{Y_{k-1}}$	E_k	W_k	C_k	L_k	L_{k-1}
$\sqrt{Y_{k-1}}$	1	-0.205	0.083	0.329	-0.248	-0.075
E_k	-0.205	1	0.18	-0.201	-0.183	0.073
W_k	0.083	0.18	1	0.3	0.026	0.049
C_k	0.329	-0.201	0.3	1	-0.102	-0.101
L_k	-0.248	-0.183	0.026	-0.102	1	0.082
L_{k-1}	-0.075	0.073	0.049	-0.101	0.082	1

TABLE 4.6: *Pearson correlation coefficients of regression (4.2).*

Pearson correlation coefficients describes the relationship between two different independent variables, there should be no multicollinearity in independent variables. Table 4.6 indicates the relationship between any two independent variables are not above 0.8 or below -0.8, it is assumed that no multicollinearity is present in the independent variables and Assumption 5 holds. In conclusion, the model is a good fit for the data, regression equation (4.2) makes intuitive sense and all assumptions of multiple regression are satisfied.

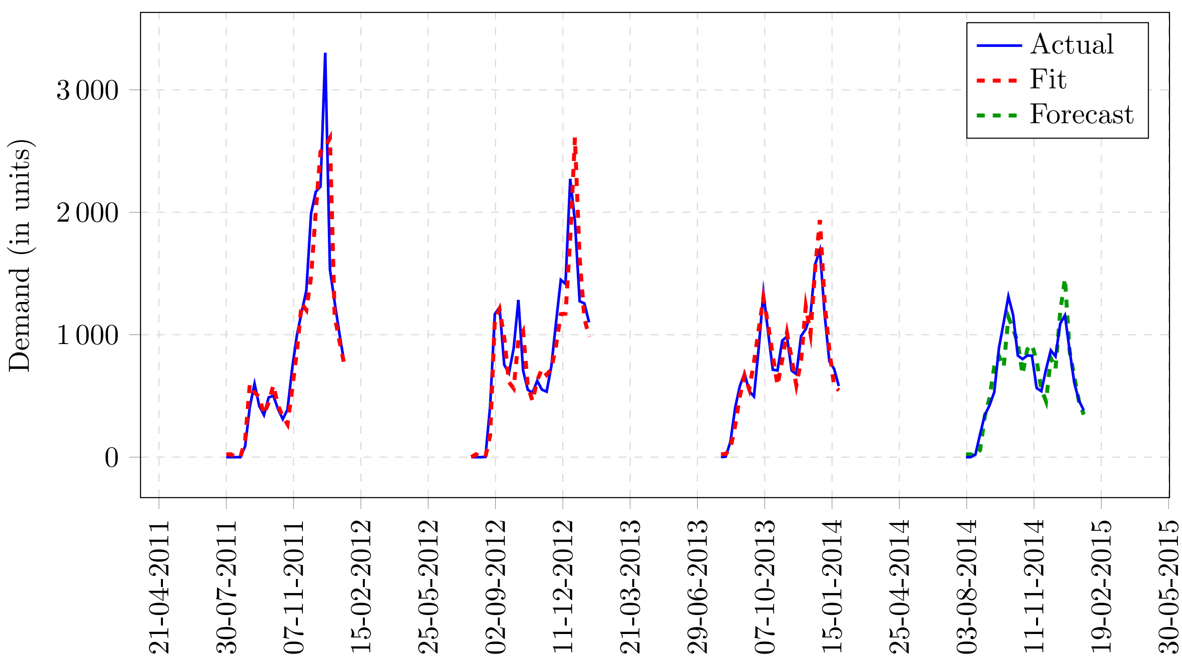

FIGURE 4.2: *Graphical display of Subclass S2 fit and forecast accuracy obtained via regression (4.2) for the years 2011–2014.*

A graphical representation of the fit and forecast accuracy is presented in Figure 4.2. The overall fit and forecast of the data is close to the actual demand as estimated and calculated from historical sales data. Regression equation (4.2) overestimates predicted demand for the two weeks surrounding Christmas, as demand for this time period was historically higher compared to actual demand in 2014. The Retailer's total actual demand for 2014 is 17 352 units and the predicted demand is 15 604 units, resulting in a 10.07% underestimation.

4.3.3 Subclass W1 regression validation

Based on the pattern of estimated demand data for the winter subclasses considered in this study, the following variables are defined for inclusion in the model. Let $\mathcal{K} = 1, 2, \ldots, k, \ldots, K$ be the set of weeks in a season, and let

Y_k be the total estimated demand in week k,
L_k be the total unit inflow during week k as planned by the Retailer,

$$E_k = \begin{cases} 1 & \text{if the last day of week } k \text{ is after the } 29^{\text{th}} \text{ or before the } 11^{\text{th}} \text{ of a month,} \\ 0 & \text{otherwise,} \end{cases}$$

$$F_k = \begin{cases} 1 & \text{if week } k \text{ falls in Febuary,} \\ 0 & \text{otherwise,} \end{cases}$$

$$M_k = \begin{cases} 1 & \text{if week } k \text{ falls in March,} \\ 0 & \text{otherwise,} \end{cases}$$

$$A_k = \begin{cases} 1 & \text{if week } k \text{ falls in May,} \\ 0 & \text{otherwise,} \end{cases}$$

$$J_k = \begin{cases} 1 & \text{if week } k \text{ falls in June,} \\ 0 & \text{otherwise,} \end{cases}$$

$$U_k = \begin{cases} 1 & \text{if week } k \text{ falls in July,} \\ 0 & \text{otherwise, and let} \end{cases}$$

$$G_k = \begin{cases} 1 & \text{if week } k \text{ falls in August,} \\ 0 & \text{otherwise.} \end{cases}$$

The dependent variable, Y_k, is the estimated demand in week k, and $\hat{Y}_k$ is the forecasted weekly demand. As for summer subclasses, variable L_k is included to account for anticipated demand, as planned by the Retailer, in the form of total weekly planned inflow in week k, and E_k is included to account for monthly demand, which—as with summer subclasses—is higher due to salaries and wages. Initial experiments indicated the presence of positive autocorrelation, and a weekly lag of demand and inflows were included to remove autocorrelation in the model. The model must be applicable to Subclass W2, on inspection of W2 heteroscedasticity is an issue for W2. To ensure the model is homoscedastic, $\sqrt{\hat{Y}_k}$ and $\sqrt{Y_{k-1}}$ replace $\hat{Y}_k$ and Y_{k-1}.

The winter selling season spans from the beginning of February until the end of July, lasting exactly 26 weeks. The pattern of estimated demand indicated that April has the highest recorded demand, possibly due to Easter festivities. Monthly dummy variables were included in this model, where April is kept as the reference month.

Having identified the variables, parameters $\hat{\beta}_1, \hat{\beta}_2, \ldots, \hat{\beta}_{10}$ were estimated and the resulting regression equation for Subclass W1 is given by

$$\sqrt{\hat{Y}_k} = 39.99 + 0.57\sqrt{Y_{k-1}} + 24.42E_k - 27.66F_k - 24.06M_k - 10.42A_k - 11.86J_k \\ -26.46U_k - 76.91G_k + 0.00036L_k + 0.00029L_{k-1}. \qquad (4.3)$$

Regression equation (4.3) makes intuitive sense, when all variables are 0, the week falls in the month of April and demand is expected to be positive. Demand is higher in week k if positive demand occurred last week $k-1$. The high coefficient of E_k is reasonable for the same reason as identified in the summer subclasses (that demand is higher after the Retailer's customers receive salaries and wages, usually paid at the end of the month). Monthly dummy variable coefficients confirm demand during April is indeed highest, as all signs for these dummy variables are negative and the coefficient value indicates the magnitude of difference between expected demand during April and any of the other months. The second highest demand is in May, followed by June, March, July, February and lastly (demand is expected to be smallest in) August. Positive inflows this week and last week are likely to increase demand this week as the coefficients are positive but relatively small compared to the other variable coefficients.

The coefficient of determination indicates a good fit, R^2=0.8471 and adjusted R^2=0.8243. The joint explanatory power of independent variables are tested through the F-test and corresponding p-value which is smaller than 0.05, concluding the model as a whole is statistically significant. The significance of each independent variable is reported on in Table 4.7. Two variables, A_k and L_{k-1} are not statistically significant (p-value > 0.05), the remaining variables are all statistically significant. Demand during May, A_k, is not significantly different from demand during April, however it may be significantly different from demand during other months. The lag variable, $\sqrt{Y_{k-1}}$ is sufficient in removing autocorrelation in this dataset. To ensure the model is applicable to other datasets where autocorrelation may be a bigger issue, L_{k-1} is kept even though it is not statistically significant for this dataset.

Variable	t value	p value
Intecept	3.84	0.0003
$\sqrt{Y_{k-1}}$	7.32	$<$ .0001
E_k	6.61	$<$ .0001
F_k	-3.10	0.0028
M_k	-3.58	0.0006
A_k	-1.77	0.081
J_k	-2.06	0.043
U_k	-3.95	0.0002
G_k	-4.91	$<$ .0001
L_k	2.28	0.0261
L_{k-1}	1.96	0.054

TABLE 4.7: *Parameter estimates for regression equation (4.3), t-values and p-values.*

The assumptions of multiple regression are formally tested for regression (4.3). All coefficients in Subclass W1's regression equation are constant and there is linearity in parameters, so Assumption 1 holds. Assumption 2 of homoscedasticity in the residuals is formally tested using Breusch-Pagan test. The p-value is 0.4897 (as obtained from SAS 9.4 [36]) for regression equation (4.3), the null hypothesis is not rejected and residuals are assumed to be homoscedastic. Concluding that Assumption 2 holds.

Results obtained via SAS 9.4 [36] for four tests of normality are presented in Table 4.8. The null hypothesis for each test is that residuals follow a normal distribution. The p-values for all tests are greater than 0.05, the level of significance; and the null hypothesis is not rejected. Assumption 3, which states that all residuals are normally distributed, holds.

Equation (4.3) contains an intercept and the assumption of Durbin-Watson is valid. The Durbin-Watson test statistic is 2.01 (obtained via SAS 9.4 [36]), the lower and upper bounds are 1.39

Test	Statistic	p value
Shapiro-Wilk	0.98	0.40
Kolmogorov-Smirnov	0.046	> 0.15
Cramer-von Mises	0.024	> 0.25
Anderson-Darling	0.24	> 0.25

TABLE 4.8: *Statistical test for normality of Subclass W1.*

and 1.9 respectively ($n = 77$, df $= 10$, $\alpha = 0.05$). As the test statistic is larger than the upper bound, the null hypothesis of no positive or negative autocorrelation is not rejected and it is assumed no autocorrelation is present in the residuals, Assumption 4 holds. To test for multicollinearity, the Pearson correlation coefficient—presented in Table 4.9—indicate no linear relationship between any two different independent variables and Assumption 5 holds.

	$\sqrt{Y_{k-1}}$	E_k	F_k	M_k	A_k	J_k	U_k	G_k	L_k	L_{k-1}
$\sqrt{Y_{k-1}}$	1	-0.158	-0.614	-0.287	0.328	0.241	-0.016	-0.084	-0.119	0.133
E_k	-0.158	1	-0.08	0.087	-0.029	0.004	-0.029	0.177	0.132	-0.015
F_k	-0.614	-0.08	1	-0.186	-0.174	-0.18	-0.174	-0.055	0.089	-0.06
M_k	-0.287	0.087	-0.186	1	-0.209	-0.216	-0.209	-0.066	0.273	0.166
A_k	0.328	-0.029	-0.174	-0.209	1	-0.202	-0.195	-0.062	0.08	0.048
J_k	0.241	0.004	-0.18	-0.216	-0.202	1	-0.202	-0.064	-0.1	-0.043
U_k	-0.016	-0.029	-0.174	-0.209	-0.195	-0.202	1	-0.062	-0.296	-0.257
G_k	-0.084	0.177	-0.055	-0.066	-0.062	-0.064	-0.062	1	-0.102	-0.101
L_k	-0.119	0.132	0.089	0.273	0.08	-0.1	-0.296	-0.102	1	0.12
L_{k-1}	0.133	-0.015	-0.06	0.166	0.048	-0.043	-0.257	-0.101	0.12	1

TABLE 4.9: *Pearson correlation coefficients of regression (4.3).*

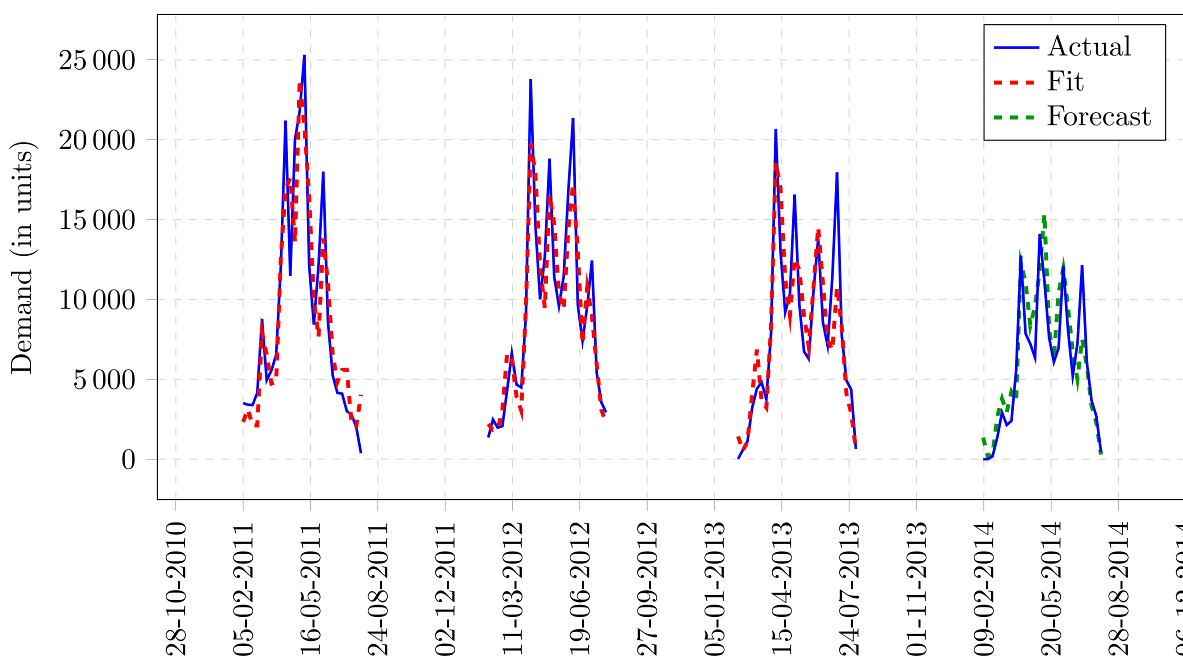

FIGURE 4.3: *Graphical display of the fit and forecast accuracy of Subclass W1, obtained via regression (4.3) for the years 2011–2014.*

Total actual demand for W1 as estimated from historical sales data for 2014 is equivalent to 152 202 units. The regression equation (4.3) overestimates demand for this period by 26.31%, predicting 192 247 units of demand for 2014. A graphical representation of the fit (red, dashed) and forecast (green, dashed) accuracy generated via regression (4.3) indicated the overestimation of demand is around April and an underestimation in July. All the assumptions of multiple regression hold and regression equation (4.3) is an acceptable model to forecast weekly demand for Subclass W1.

4.3.4 Subclass W2 regression validation

The regression model developed for Subclass W1 in §4.3.3 as the training set is validated for Subclass W2. The variables defined for inclusion in Subclass W1's regression equation (4.3), when applied to Subclass W2 is given by

$$\begin{aligned}\sqrt{\hat{Y}_k} = 4.8 + 0.81\sqrt{Y_{k-1}} + 5.48E_k - 5.93F_k - 2.13M_k - 1.76A_k + 0.29J_k \\ -6.92U_k - 17.79G_k + 0.0011L_k + 0.0009L_{k-1}.\end{aligned} \tag{4.4}$$

The relative values of the coefficients in regression equation (4.4) are as expected and similar to equation 4.3, apart from dummy variable for June, J_k, which is positive as opposed to negative. The positive sign indicates demand in the month of June is likely to be higher than the chosen reference month, April. One possible reason for this is that the average temperatures in June are lower than in April, as this subclass offers warm jackets to end consumers, a higher demand in the cooler months is expected. All coefficients are smaller than in regression equation (4.3), because total demand for this product is lower than Subclass W1.

The coefficient of determination indicates a good fit, R^2=0.9474 and adjusted R^2=0.9395. The p-value of the F-test, obtained via SAS 9.4 [36], is smaller than 0.0001 indicating the independent variables joint explanatory power is statistically significant. To test the significance of each independent variable from regression equation (4.4), a t-test was performed. Table 4.10 provides the results of each variables t-test and corresponding p-values as obtained via SAS 9.4 [36]. The intercepts p-value is larger than 0.05, meaning there could be significant indicators in the model that have not been taken into account. Variables U_k and G_k are statistically significant, indicating the need for monthly dummy variables as demand during July and August are significantly different from demand during April, the reference month. Monthly dummy variables: F_k, M_k, A_k and, J_k are not statistically significant. These variables are not significantly different from demand during April, however they may be significantly different from demand in other months (from one another) and thus remain in the model. The remaining independent variables $\sqrt{Y_{k-1}}$, E_k, L_k and, L_{k-1} are highly significant, p-value < 0.0001.

Overall the model is a good fit and the validity of the model must be tested by checking that the assumptions of multiple regression, as listed in §3.4.2. hold. All the coefficients in regression equation (4.4) are constant and the equation is linear in its parameters so Assumption 1 holds.

The second assumption is that residuals are homoscedastic. Assumption 2 is formally tested using the Breusch-Pagan test, where the null hypothesis of homoscedasticity is tested against the alternative hypothesis of heteroscedasticity. The resulting p-value of the Breusch-Pagan test is smaller than 0.05, the level of significance. Therefore, the null hypothesis is rejected in favour of the alternative hypothesis (assumed heteroscedasticity). If the level of significance is $\alpha = 0.01$ for this test, the null hypothesis would not be rejected (assumed homoscedasticity). To obtain more certainty of homoscedasticity in residuals, White's test is performed. The null hypothesis for White's test is that the variances for the errors are equal (homoscedastic) and the

Variable	t value	p value
Intercept	1.56	0.123
$\sqrt{Y_{k-1}}$	16.81	$< .0001$
E_k	4.04	0.0001
F_k	-1.76	0.0838
M_k	-0.83	0.4119
A_k	-0.73	0.4661
J_k	0.12	0.9021
U_k	-2.74	0.0078
G_k	-2.92	0.0048
L_k	5.09	$< .0001$
L_{k-1}	4.29	$< .0001$

TABLE 4.10: *Parameter estimates for regression equation (4.4), t-values and p-values.*

alternative hypothesis is that the variances of the errors are not equal (heteroscedastic). The resulting p-value (obtained via SAS 9.4 [36]) of White's test is 0.1037, at a significance level of $\alpha = 0.05$, the null hypothesis is not rejected and homoscedasticity is assumed, concluding Assumption 2 holds.

Test	Statistic	p value
Shapiro-Wilk	0.97	0.079
Kolmogorov-Smirnov	0.09	0.089
Cramer-von Mises	0.10	0.13
Anderson-Darling	0.67	0.08

TABLE 4.11: *Statistical test for normality of Subclass W2.*

Four test of normality obtained via SAS 9.4 [36] are presented in Table 4.11 and indicate whether the residuals follow a normal distribution. The p-values for all four tests are greater than 0.05 and the null hypothesis is not rejected. It is assumed that the residuals follow a normal distribution and Assumption 3 holds.

To test for autocorrelation amongst residuals the Durbin-Watson test statistic is computed. The lower and upper bounds are 1.38 and 1.9 respectively ($n = 77$, df $= 10$, $\alpha = 0.05$). As the computed Durbin-Watson test statistic lies between the bounds ($1.38 <$ DW$= 1.823 < 1.9$), the test is inconclusive.

The final assumption of multicollinearity is tested for equation 4.4. There is no signal of multicollinearity in the information so far, the signs and size of the regression coefficients are reasonable and most of the p-values are significant. The pairwise correlation coefficients between variables are reported on in Table 4.12. There is no indication that multicollinearity plays a significant role as correlation coefficients between any two different variables are neither above 0.8 or below -0.8.

The fit and forecast accuracy of regression equation (4.4) in Figure 4.4 indicates an overestimation of demand during the month of June, followed by an underestimation in July. The actual demand, as estimated and calculated from historical sales; in 2014 amounts to 72 472 units and the total predicted demand for the same time period is 77 693 units, resulting in a 7.2% overestimation of demand from equation 4.4.

	$\sqrt{Y_{k-1}}$	E_k	F_k	M_k	A_k	J_k	U_k	G_k	L_k	L_{k-1}
$\sqrt{Y_{k-1}}$	1	-0.075	-0.634	-0.316	0.339	0.345	0.097	-0.039	0.024	0.229
E_k	-0.075	1	-0.08	0.087	-0.029	0.004	-0.029	0.177	-0.05	0.021
F_k	-0.634	-0.08	1	-0.186	-0.174	-0.18	-0.174	-0.055	-0.111	-0.195
M_k	-0.316	0.087	-0.186	1	-0.209	-0.216	-0.209	-0.066	0.079	0.105
A_k	0.339	-0.029	-0.174	-0.209	1	-0.202	-0.195	-0.062	0.07	0.088
J_k	0.345	0.004	-0.180	-0.216	-0.202	1	-0.202	-0.064	-0.087	0.054
U_k	0.097	-0.029	-0.174	-0.209	-0.195	-0.202	1	-0.062	-0.246	-0.243
G_k	-0.039	0.177	-0.055	-0.066	-0.062	-0.064	-0.062	1	-0.079	-0.079
L_k	0.024	-0.05	-0.111	0.079	0.07	-0.087	-0.246	-0.079	1	0.273
L_{k-1}	0.229	0.021	-0.195	0.105	0.088	0.054	-0.243	-0.079	0.273	1

TABLE 4.12: *Pearson correlation coefficients of regression (4.4).*

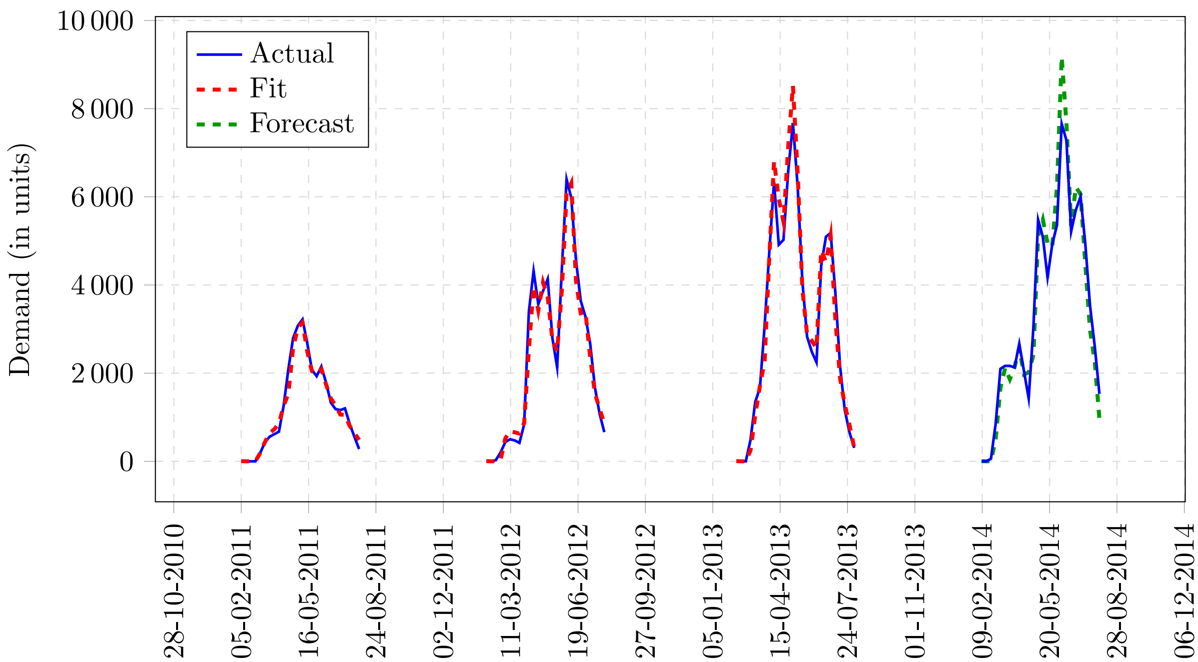

FIGURE 4.4: *Graphical display of the fit and forecast accuracy for Subclass W2, obtained via regression (4.4) for the years 2011–2014.*

4.4 Black-box validation

In black-box validation, the internal procedures of the model are disregarded and only the overall behaviour of the model is considered. Confidence in the model is gained when, given the same inputs as the real world system, the observed outputs are sufficiently similar [34].

An existing simulation model of the Retailer was built for a similar purpose of simulating weekly sales, on a company level (for all stores, and all sizes) in Thom [40]. The Retailer's real inflows were used as input and simulated sales were compared to the Retailer's sales using the intraclass correlation coefficient (ICC) statistic. Typically, ICC is used to assess the reliability between two or more systems recorded quantitative outcomes for a group of values. In this case, two systems: the Retailer's real sales and the simulation model outcomes which are grouped into three groups: weeks, stores and, sizes. All ICC values are close to 1, indicating a very high correlation between actual and simulated sales, validating the model to be sufficiently close to the real system [40].

Total sales generated by the existing model are used alongside the Retailer's total sales in the black-box validation of sales generated by the proposed simulation model, built to facilitate dynamic size profile adjustments. Total sales generated by the simulation model in this study (proposed model) are expected to resemble the Retailer's actual sales (real system) and the validated existing model sales (existing model). The proposed model is valid if it generates sales as expected. Reliability of the proposed model is assessed by a comparison of sales generated by the real system.

The proposed simulation model, which enables dynamic size profile adjustments; generates sales weekly for the company (all stores, and all sizes). To test the black-box validity of the proposed simulation model, a weighting parameter, γ, is set to be 1 and used in the dynamic adjustment of size profiles. The use of $\gamma = 1$ ensure the Retailer's calculated size profile does not adjust throughout the season, thus testing whether output generated by the proposed model is sufficiently accurate for the purpose of this study.

Inter-rater reliability assesses the degree to which different systems (real system and proposed model) give consistent estimates of the same observation. To test the consistency of total sales between the real system and the proposed model, an ICC based on a two-way analysis of variance (ANOVA) with random effect is performed for each group of sales (weeks, stores, and sizes). All ICC values are obtained via IBM SPSS 25 [16]. Two scores of the total sales are presented per group, the first is a single score ICC which indicates the reliability of a single observation, measuring consistency between the two systems (real system and the proposed simulation model) of single sales observations in a group. The second, is an average score ICC which indicates reliability of observations on average, measuring how consistent the two systems (real system and the proposed model) are to each other, on average.

The simulation model is stochastic, thus output from one simulation run may be different from another. Confidence in the model results can be improved by performing multiple replications [34]. A formula based on a t-confidence interval for the estimate of mean μ from m initial replications may be used to calculate the necessary number of simulation replications, given the output from simulation replications are independent, identically and normally distributed [3]. Given m initial replications, let

$N(m)$	be the number of replications required,
$\bar{X}(m)$	be the estimate of the mean μ,
$S(m)$	be the estimate of the standard deviation σ,
α	be the level of significance used,
ϵ	be the allowable percentage error for the estimate $\bar{X}(m)$, where $\epsilon = \lvert\bar{X}(m) - \mu\rvert / \lvert\mu\rvert$, and let
$t_{m-1,1-\alpha/2}$	be the critical value of the two-tailed t-distribution at a significance level of α, given $m-1$ degrees of freedom.

Then $N(m)$, the number of replications required, is given by

$$N(m) = \left(\frac{S(m)t_{m-1,1-\alpha/2}}{\bar{X}(m)\epsilon}\right)^2. \tag{4.5}$$

Total sales from one replication do not influence the output of another replication, therefore simulation replications are independent. Thom [40] confirmed normality of simulated sales output by performing four formal tests of normality, namely the Shapiro-Wilk, Kolmogorov-Smirnov, Cramer-von Mises and Anderson-Darling tests on 100 values of total sales, generated using actual inflow. Based on the validity of the existing model and restricted by lengthy simulation solution time, ten replications of the proposed simulation model were run in Python 3.6.3 [32] to test normality. Inflows are calculated using the size-mix allocation (available in §3.2.2), where $\gamma = 1$ was used in the adjustment algorithm (meaning no size profile adjustments actually occurred/the Retailer's size profile remains static).

Due to the small sample size ($n = 10$), two formal tests of normality (Shapiro-Wilk and Anderson-Darling) test the null hypothesis which states that the difference in means follows a normal distribution against the alternative hypothesis which states the data are not normally distributed. The results as obtained via Python 3.6.3 [32] are given in Table 4.13.

Test	Statistic	p value
Shapiro-Wilk	0.941	0.567
Anderson-Darling	0.277	0.684

TABLE 4.13: *Results of tests for normality on simulation model output (total sales), generated for Subclass S1.*

At a significance level of $\alpha = 0.05$, the p-value as obtained for both tests of normality are greater than 0.05. As a result the null hypothesis is not rejected and assuming normality of the output, the necessary number of replications may be calculated using formula (4.5). As the same simulation logic is applied to all subclasses considered in this study, the output from simulation replications in all subclasses are independent, identically and normally distributed.

4.4.1 Subclass S1 model validation

The proposed simulation model generates weekly sales for Subclass S1 during 2014, on a store and size level. Input parameters of demand are generated via an underlying regression equation (equation (4.1)) which is considered a good fit for the data and validated in §4.3.1. Stock inflow quantities are calculated iteratively throughout the simulation, when stock arrives at the DC; using a size-mix allocation algorithm which is validated to be sufficiently close to the Retailer's actual method of allocation. The inflows are calculated in this manner to enable dynamic size profile adjustments, where a weighting parameter, $\gamma = 1$ is used to ensure the Retailer's size profile does not adjust.

To examine the validity and reliability of the proposed simulation model for Subclass S1, sales are analysed and compared to sales recorded in the real system and an existing model (validated in a previous study to have sufficient reliability when real inflows are used as input [40]). Using formula 4.5, at least one replication of the simulation model is required for S1. However, due to availability of computer resources, the proposed model is replicated 10 times to increase confidence in the models reliability.

Table 4.14 presents two ICC scores based on a two-way ANOVA with random effects, for groups of total sales recorded for weeks, stores and sizes as obtained via IBM SPSS 25 [16]. The sales generated by the proposed simulation model are compared to the real systems sales for each of these groups to assess reliability of recorded sales between the two systems. The single ICC assesses the consistency between the systems for each single sale value, while the average ICC assesses the consistency between the two systems recorded sales, on average. Cronbach's α is a coefficient of internal reliability, an α value close to 1 indicates high covariance amongst items which likely measure the same underlying concept.

Each group of single and average ICC values are greater than 0.9, indicating excellent reliability

Group	Single measure ICC	Average measure ICC	Cronbach's Alpha (α)
Week	0.942	0.970	0.969
Store	0.949	0.947	0.974
Size	0.996	0.998	0.998

Table 4.14: *Groups of ICC values for Subclass S1 sales generated via the proposed simulation model, compared to the Retailer's real sales. A value of $\gamma = 1$ was used.*

of the proposed simulation model in generating sales for each level (weeks, stores, and sizes). The high ICC values specify that there is not a lot of variability between the two systems (real system and proposed model), and the α value close to 1 indicates excellent internal reliability for each group. The proposed simulation model is considered reliable in the ability to generate sales which are consistent with the real systems sales.

Validity of the proposed simulation model is assessed by determining whether total sales are generated as expected. Total sales generated by the proposed model for Subclass S1 are 89 084.3 units, on average with a standard deviation of 215.78 units.

In 2014 the Retailer recorded 88 581 unit sales in total for S1. For the same year, the validated existing model simulated a total of 90 139.4 units of sale, on average. The sales generated by the proposed model are 0.57% more than sales recorded in the real system and 1.17% less than sales generated by the existing model. As total sales generated by the proposed model are between the real system sales and the existing model sales, and all ICC values indicate excellent reliability (> 0.9), the proposed simulation model is confirmed to be a valid and reliable representation of the real systems sales for Subclass S1.

Graphical comparison of data

Scatter plots of the real system, and proposed model total sales provide a visual representation of the correlation between systems. A line of best fit (red solid line) is included to represent the trend of total sales between systems. Points lying around the line indicate a correlation between systems, meaning the relationship between sales generated by the proposed simulation model and sales recorded by real system are as expected.

The scatter plot of total sales for weeks are presented in Figure 4.5, all the points are close to the line of best fit and coefficient of determination, $R^2 = 0.8938$, indicating sales generated weekly by the simulation model are as expected.

Figure 4.6 presents the correlation between real system sales for stores, against sales generated by the simulation model for stores. The coefficient of determination for stores is $R^2 = 0.8998$, which states the predicted total sales on a store level are a good fit for the actual sales recorded by the Retailer at stores in Subclass S1. An outlier at point (A), indicates an overestimation of sales for this store (Store 734). The overestimation of sales at this store is due to a 0.0034 increase in predicted demand at this store for 2014, compared to the actual proportion of demand as estimated (0.0014). Points close to the identified outlier, are affected similarly by their corresponding demand prediction. The majority of stores sell less than 250 units of S1 for the season, this is evident by the cluster of points in the lower left-hand corner of the scatter plot.

Points in Figure 4.7 represent a correlation between total sales on a size level for the Retailer's actual sales against the simulation model's sales. All points lie close to the line of best fit (red solid line) and $R^2 = 0.9942$, indicating the simulation model is able to generate sales on a size

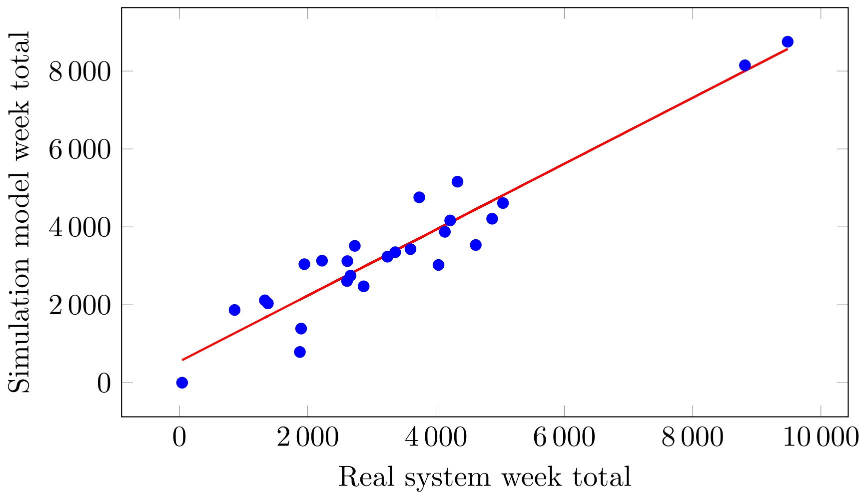

FIGURE 4.5: *Scatter plot of correlation between total real system week sales and total simulated week sales for Subclass S1. The simulation was performed using* $\gamma = 1$.

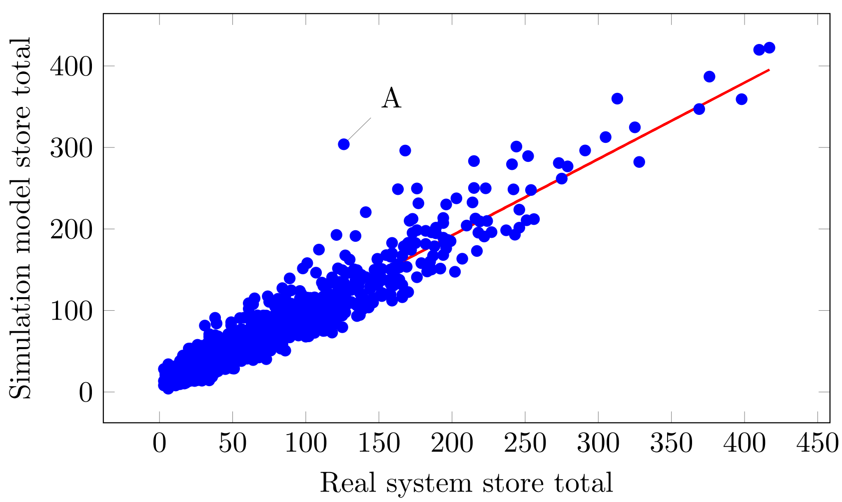

FIGURE 4.6: *Scatter plot of correlation between total real system store sales and total simulated store sales for Subclass S1. The simulation was performed using* $\gamma = 1$.

level that closely represent the sales as expected

The scatter plots presented in this section, provide additional support of the simulation model's ability to generate sales for Subclass S1 as expected, confirming validity and concluding the black box validation.

4.4.2 Subclass S2 model validation

In this section, black-box validation of sales generated by the proposed simulation model for Subclass S2 are assessed. Input parameters of demand are generated using an underlying regression equation (equation (4.2)), defined and validated in §4.3.2 to meet all the assumptions of multiple regression and be a good fit of the data. Inflows are calculated by a size-mix allocation calculation which is shown in a previous study to not be significantly different from the Retailer's calculated inflows [40]. The weighting parameter for dynamic size profile adjustments, $\gamma = 1$, are used to ensure no adjustment to the Retailer's calculated size profile occurs and the

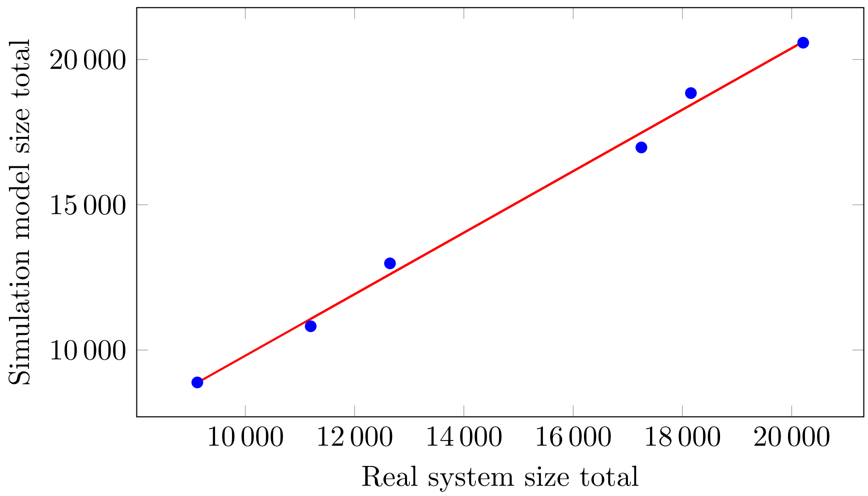

FIGURE 4.7: *Scatter plot of correlation between total real system size sales and total simulated size sales for Subclass S1. The simulation was performed using $\gamma = 1$.*

simulation model is reflective of a static size profile system. To test reliability and validity, the proposed simulation model total sales were compared against the real systems total sales and an existing models generated total sales, along with ICC from a two-way random ANOVA of weeks, stores and sizes sales against the real systems sales.

Using formula (4.5), at least three replications of the proposed model are required for S2. Given sufficient available computer resources, the proposed model was replicated 10 times and the average output was reported on. According to the ICC values obtained via IBM SPSS 25 [16], presented in Table 4.15, a very good reliability between the real sales and simulated sales exists for each group. Each group's α is greater than 0.9, indicating very high covariance between the systems (real system and proposed model).

Group	Single measure ICC	Average measure ICC	Cronbach's Alpha (α)
Week	0.893	0.943	0.952
Store	0.920	0.958	0.961
Size	0.897	0.946	0.979

TABLE 4.15: *Groups of ICC values for Subclass S2 sales generated via the proposed simulation model, compared to the Retailer's real sales. A value of $\gamma = 1$ was used.*

On average, total sales generated by the proposed simulation model for Subclass S2 are 12 688.4 units, with a standard deviation of 84.75 units. The Retailer's recorded actual sales for S2 are 14 250 units and total sales generated by the existing model are 13 220.9 units, on average. The proposed model output is 10.96% less than the real system sales as recorded by the Retailer, and 4.03% less than the existing model sales.

Throughout the season, Subclass S2 receives just three styles from factories, in weeks 4, 9 and 18. A possible reason for the decrease in expected sales arises from the assumption of store allocation, which states that stores receive stock on the same day as it arrives at the DC from factories. To enable the simulation model to dynamically adjust size profiles, the most recent available sales data was needed at the time of allocation. Thereafter, the adjusted size profile is used to calculate the size-mix allocation. The Retailer's lead time from when stock arrives at the DC until it is available in stores varies amongst stores, with no indication of the style of stock a store received. Analysis of the historical data available from the Retailer was unable to determine why inflow distribution varied amongst stores and there was no pattern in the inflows arrival.

However, given that the proposed simulation total sales are only about 4% less than the validated existing model and the excellent reliability of single store ICC and average ICC measures (> 0.9) for each group, and good single week and size ICC values (> 0.89) the proposed simulation model is considered valid and reliable for the purpose of this study in generating sales for S2.

Graphical comparison of data

Scatter plots of total sales for the weeks are presented in Figure 4.8, with a line of best fit (red solid line) indicating the correlation between systems total sales. The coefficient of determination, $R^2 = 0.8509$ states that the predicted sales are a good fit for the actual sales in this subclass. There are a few points below the line of best fit, indicating an underestimation of total weekly sales by the simulation model. Inflows are calculated in the weeks that stock arrives at the DC from factories and there are only three styles planned to arrive for Subclass S2 for 2014. An assumption of the simulation model which may explain the lower simulated sales amount is that no carry over stock from the previous season is included. However, in the real system carry over stock remains in the system and sales are recorded without style information. The outliers occur at the season's start, thus concluding that increased sales recorded by the real system are attributed to a proportion of carry over stock that is sold, but unable to be removed as no information regarding specific style sales is included in the data.

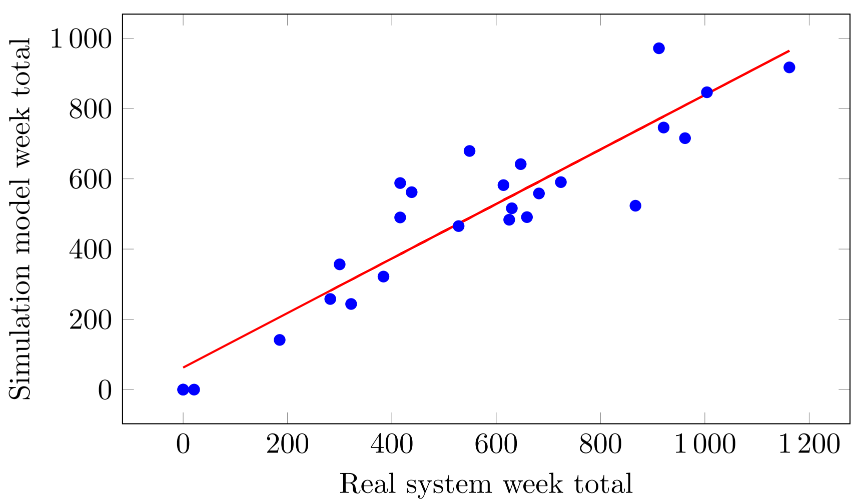

FIGURE 4.8: *Scatter plot of correlation between total real system week sales and total simulated week sales for Subclass S2. The simulation was performed using* $\gamma = 1$.

The inflows calculated weekly are assumed to be sent to stores on the same day as stock arrives at the DC from factories, meaning there is a zero lead time of shipping from DC to stores. In the real system, the lead time is between 2–3 weeks and each style is sent to different stores with varying lead times, inhibiting the identification of attributes relating to lead time in the real system.

Figure 4.9 shows the relationship between the real system total sales and total sales generated by the simulation model for Subclass S2, where a value $R^2 = 0.9883$ express an excellent fit of the generated sales to real system sales recorded for stores. The line of best fit indicates a close correlation between the systems, confirming that total sales on a store level are generated by the simulation model as expected. Therefore, the assumption of zero lead time for stock inflow to stores does not have a significant effect on total sales generated on a store level by the simulation model.

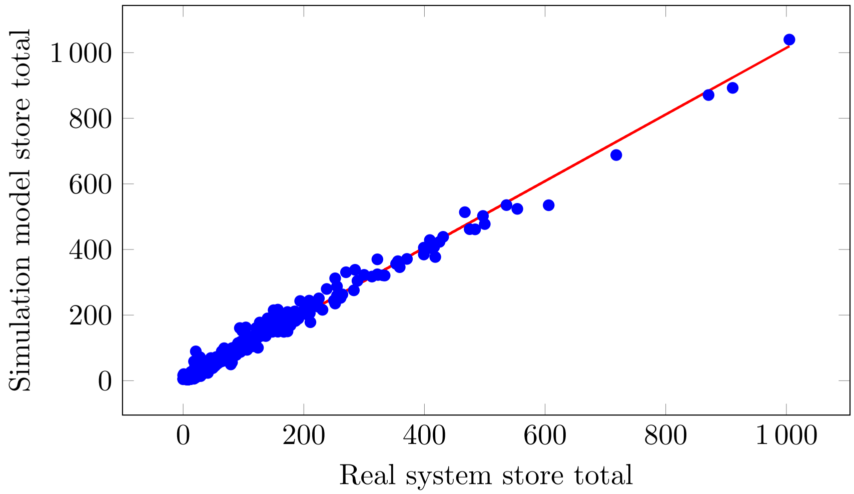

FIGURE 4.9: *Scatter plot of correlation between total real system store sales and total simulated store sales for Subclass S2. The simulation was performed using $\gamma = 1$.*

On a size level, total sales generated by the simulation model are compared with the Retailer's real sales and presented in Figure 4.10. Generally sales for the sizes in S2 are underestimated for 2014, indicated by points below the line of best fit. However, the fit of predicted sales for sizes is a good fit for the actual size sales, given by $R^2 = 0.9368$. An outlier in the top right-hand corner indicates an underestimation of total sales for the size (size 5). The real system recorded a total sale of 4 107 units for this size, compared to 3 554.6 units generated by the simulation model on average for this size due to underestimation of demand.

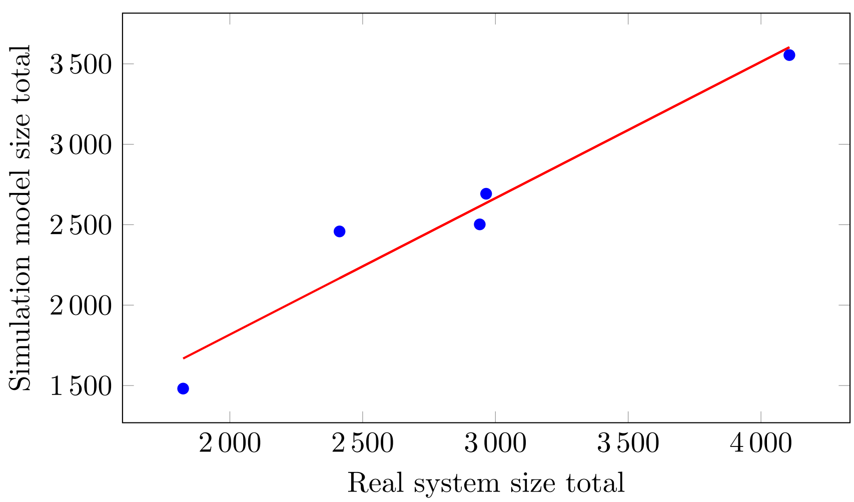

FIGURE 4.10: *Scatter plot of correlation between total real system size sales and total simulated size sales for Subclass S2. The simulation was performed using $\gamma = 1$.*

Given the high ICC and α values (all close to 1) for total sales in weeks, stores and sizes, as well as the scatter plots for these groups, the simulation model's ability to generate sales for Subclass S2 is considered valid and reliable, concluding the black-box validation.

4.4.3 Subclass W1 model validation

Input parameters of demand are generated using an underlying regression equation (equation (4.3)), defined and validated in §4.3.3 to meet all the assumptions of multiple regression and be a good fit of the data. Inflows are calculated by a size-mix allocation calculation which is shown in a previous study to not be significantly different from the Retailer's calculated inflows [40]. The weighting parameter for dynamic size profile adjustments, $\gamma = 1$, are used to ensure no adjustment to the Retailer's calculated size profile occurs and the simulation model is reflective of a static size profile system. Using formula (4.5), at least one simulation replication is required for Subclass W1. Given the availability of computer resources, the proposed model is replicated 10 times to increase confidence in the output and the average is reported on.

Reliability of the proposed simulation model is quantified by considering the ICC values, obtained via IBM SPSS 25 [16]; presented in Table 4.16 where each group and ICC measure, apart from the week single ICC (0.828); are excellent (> 0.9). The reliability of week single ICC measure is still considered good (> 0.8) and the average consistency of actual week sales and simulated week sales are excellent.

Group	Single measure ICC	Average measure ICC	Cronbach's Alpha (α)
Week	0.828	0.906	0.904
Store	0.992	0.996	0.997
Size	0.957	0.978	0.991

TABLE 4.16: *Groups of ICC values for Subclass W1 sales generated via the proposed simulation model, compared to the Retailer's real sales. A value of $\gamma = 1$ was used.*

Given the reported ICC values and the high measures of internal consistency, the proposed simulation model when generating sales for Subclass W1 are considered a reliable representation of the real system. Validity of the system must be determined, before the proposed simulation model may be used to analyse the effect of dynamic size profile adjustments.

Total sales generated by the proposed simulation model for W1 amount to 154 068.7 units, on average, with a standard deviation of 119.2 units. This value is 5.6% higher than the real systems total sales (145 904), as recorded by the Retailer and 0.31% less than the average total sales of the validated existing model (154 546.3). As the proposed simulation model total sales are within the expected values of total sales, the proposed model is considered valid and reliable with regards to objective of this study and it is concluded the effect of dynamic size profile adjustments on total sales may be analysed with confidence for Subclass W1.

Graphical comparison of data

This section presents a graphical comparison between the real system total sales, as recorded by the Retailer; and total sales generated by the proposed simulation model for Subclass W1. Scatter plots present a comparison between system sales, indicating the strength of correlation for total sales grouped by weeks, stores and sizes. Figure 4.11 presents total weekly sales as recorded by the Retailer and as generated by the simulation model for W1. There is some noticeable variation between the two systems total sales, with points lying on either side of the line of best fit. The value of R^2 is 0.6826, indicating that the relationship is not too strong, but is still acceptable. Points above the line indicate an overestimation and points below, an underestimation in total sales. An outlier indicated at point (B), is recorded for the first week

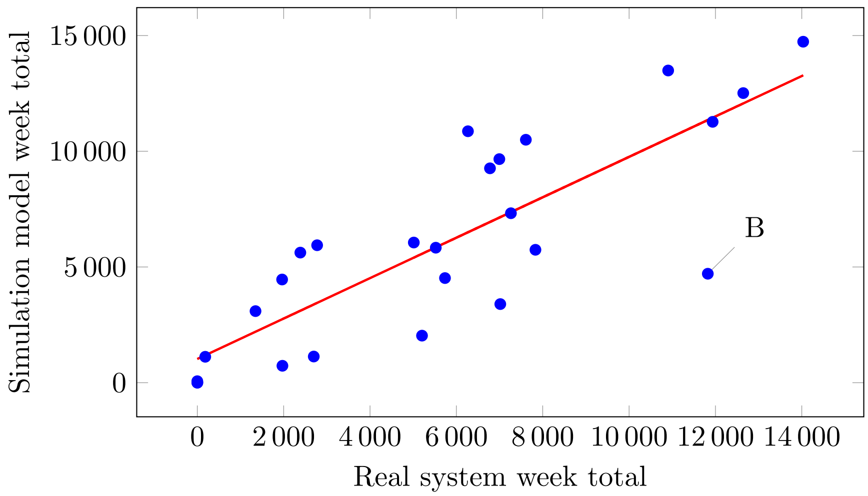

FIGURE 4.11: *Scatter plot of correlation between total real system week sales and total simulated week sales for Subclass W1. The simulation was performed using $\gamma = 1$.*

of July. Demand in July was underestimated by regression equation (4.3), which as reported in the scatter plot results in an underestimation of sales.

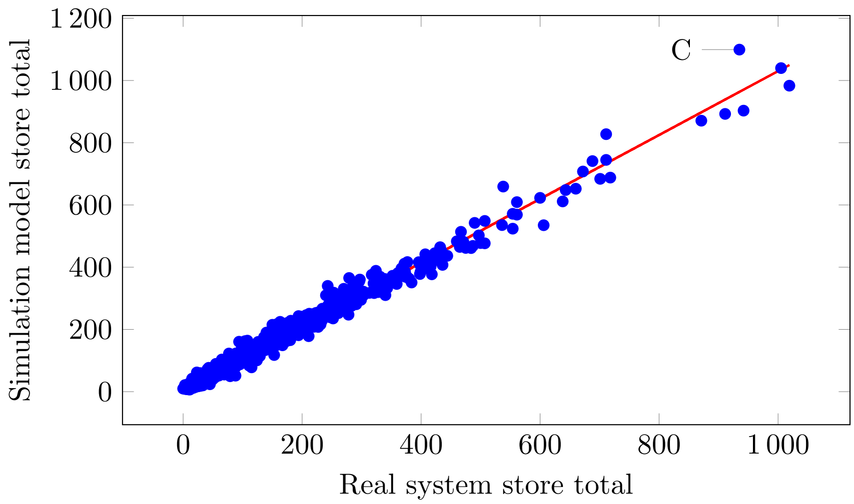

FIGURE 4.12: *Scatter plot of correlation between total real system store sales and total simulated store sales for Subclass W1. The simulation was performed using $\gamma = 1$.*

Figure 4.12 presents the relationship between total sales on a store level for the two systems under consideration (real system and proposed model). The majority of points are clustered towards the lower left-hand corner, simply indicating that the majority of stores sell under 600 units of W1 for the season, 2014. The correlation between systems is strong, indicated by the closeness of points to the line of best fit and an $R^2 = 0.9872$. An outlier at point (C), indicated an overestimation of sales for the store (Store 338). Actual sales recorded by the Retailer at this store are equivalent to 935 units for the season, which is overestimates by approximately 164 units in the simulation model.

Total sales in Figure 4.13 indicate a very good fit of sales for each size. The value of $R^2 = 0.9979$, indicates an almost perfect fit of the simulated sales to the actual sales. Total sales are overestimated because demand was overestimated by regression equation (4.3). On average the

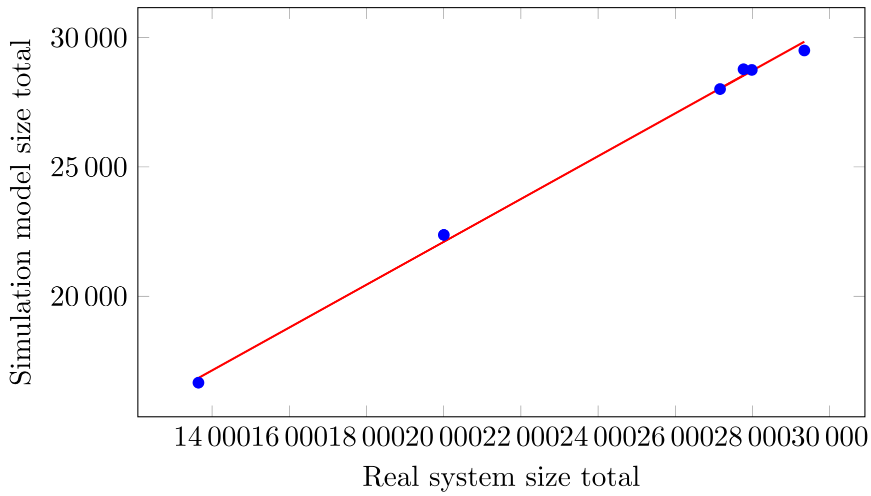

FIGURE 4.13: *Scatter plot of correlation between total real system size sales and total simulated size sales for Subclass W1. The simulation was performed using $\gamma = 1$.*

size sales generated by the simulation model are about 7% higher than actual size sales recorded by the Retailer for 2014.

Total sales generated by the proposed simulation model have a relatively strong correlation with the real system total sales for weeks, and an excellent correlation for store and size sales. Furthermore, total sales for 2014 are as expected and it is concluded that the proposed model is accepted as valid and reliable, thus completing black-box validation for Subclass W1.

4.4.4 Subclass W2 model validation

Input parameters of demand are generated using an underlying regression equation (equation (4.4)), defined and validated in §4.3.4 to meet all the assumptions of multiple regression and be a good fit of the data. Inflows are calculated by a size-mix allocation calculation which is shown in a previous study to not be significantly different from the Retailer's calculated inflows [40]. The weighting parameter for dynamic size profile adjustments, $\gamma = 1$, are used to ensure no adjustment to the Retailer's calculated size profile occurs and the simulation model is reflective of a static size profile system. Using formula (4.5), at least two replications of the proposed model are required for Subclass W2. However, due to availability of computer resources, the proposed model is replicated ten times and the average output is reported on.

Total sales generated by the proposed simulation model are assessed for reliability against the real system for each level of weekly sales, store sales and size sales using ICC measures of a two-way random ANOVA. Each of the ICC measures and corresponding α values available in Table 4.17 are obtained via IBM SPSS 25 [16].

All ICC values, apart from week single measure ICC (0.871) are excellent (> 0.9) and an indication of very little variability of sales between the real system and the proposed model is noted. Each groups high α values confirm internal consistency, a measure of correlations between different items on the same test. The resulting ICC measures for the comparison between actual system sales and the proposed simulation sales, for weeks, stores and sizes build confidence in the simulation models reliability.

Validity of the proposed simulation model is assessed by the ability to generate sales that are sufficiently close to the real system sales, as recorded by the Retailer (for the same subclass

Group	Single measure ICC	Average measure ICC	Cronbach's Alpha (α)
Week	0.871	0.931	0.933
Store	0.972	0.986	0.986
Size	0.992	0.996	0.995

TABLE 4.17: *Groups of ICC values for Subclass W2 sales generated via the proposed simulation model, compared to the Retailer's real sales. A value of $\gamma = 1$ was used.*

and season), and whether the simulation model is able to generate sales sufficiently close to the existing models total sales.

The simulation model generates, on average (after 10 replications) total sales for the season equal to 66 065.6 units, with a standard deviation of 300.68 units. Total sales recorded by the Retailer for the real system are 66 011 units, and the validated existing model's total sales are 66 697.6 units, on average. In comparison to the total sales generated by the proposed simulation model, the Retailers sales are 0.08% less than the proposed model, while the existing model generates sales that are 0.95% more than the proposed system.

Sales are as expected for the proposed model, confirming validity in the models ability to generate sales sufficiently close to the real system and existing model. In conclusion, the proposed model is accepted as a valid representation of the real system and the proposed model reliability of generating sales on a week, store and size level is excellent.

Graphical comparison of data

A graphical validation is presented in the form of scatter plots, for total sales generated by the model against the real system sales, for weeks in Figure 4.14, stores in Figure 4.15, and sizes in Figure 4.16. There are two outliers in Figure 4.14 which presents the correlation of total sales in weeks between the two systems. The points labelled (D) and (E) correspond to the first two weeks in July, where demand was underestimated by regression equation (4.4). An underestimation of demand results in an underestimation of sales by the simulation model for Subclass W2. The coefficient of determination, $R^2 = 0.7687$ indicates a strong relationship between systems for the week's total sales.

Figure 4.15 presents the relationship between store total sales as recorded in the real system, compared to values generated by the simulation model for 2014 in W2. The majority of points are in the lower left-hand corner and indicate that stores sell roughly less than 400 units of subclass W2 for the season. The correlation between systems is strong, with points lying very close to the line of best fit and an $R^2 = 0.9763$, which confirms a very good simulation of total sales on a store level.

On a size level, Figure 4.16 presents the corresponding correlation of sizes between systems. Points are close to the line of best fit, indicating a strong correlation between the real system and the proposed simulation model. Sales are generated as expected and $R^2 = 0.9927$, concluding the simulation model as an acceptable, valid and reliable method of generating sales for subclass W2 and black box validation is complete.

The effect of dynamic size profiles adjustments may be analysed with confidence for each of the subclasses considered in this study.

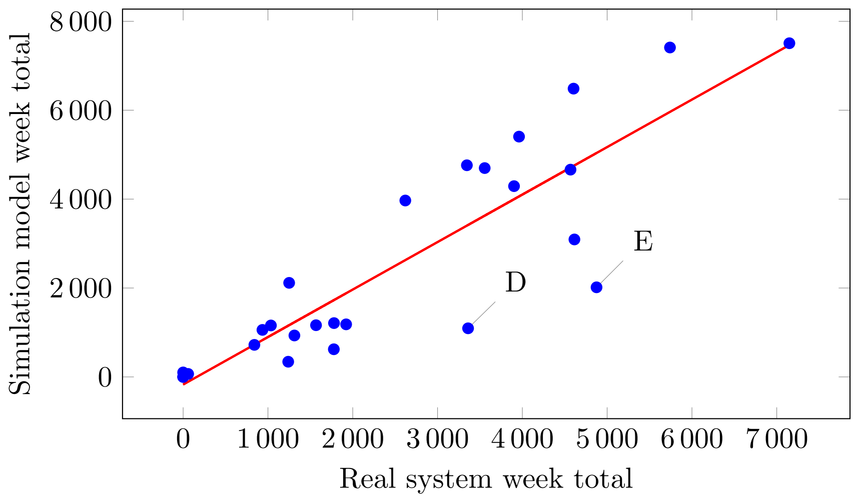

FIGURE 4.14: *Scatter plot of correlation between total real system week sales and total simulated week sales for Subclass W2. The simulation was performed using* $\gamma = 1$.

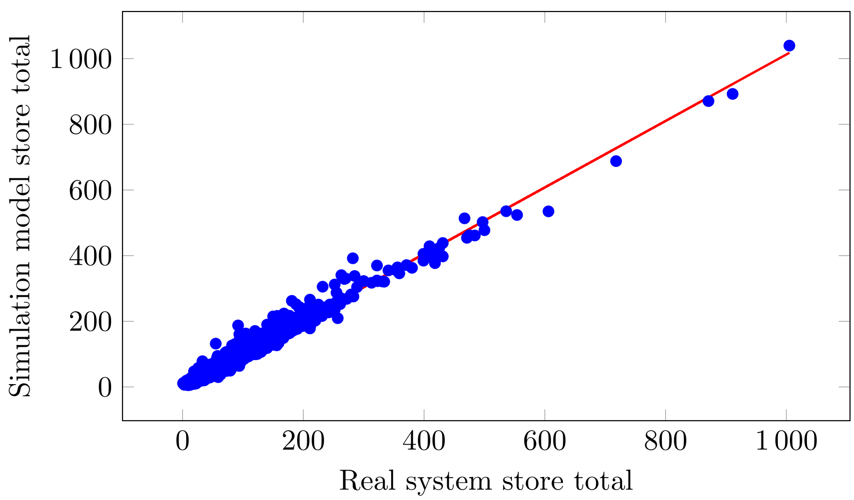

FIGURE 4.15: *Scatter plot of correlation between total real system store sales and total simulated store sales for Subclass W2. The simulation was performed using* $\gamma = 1$.

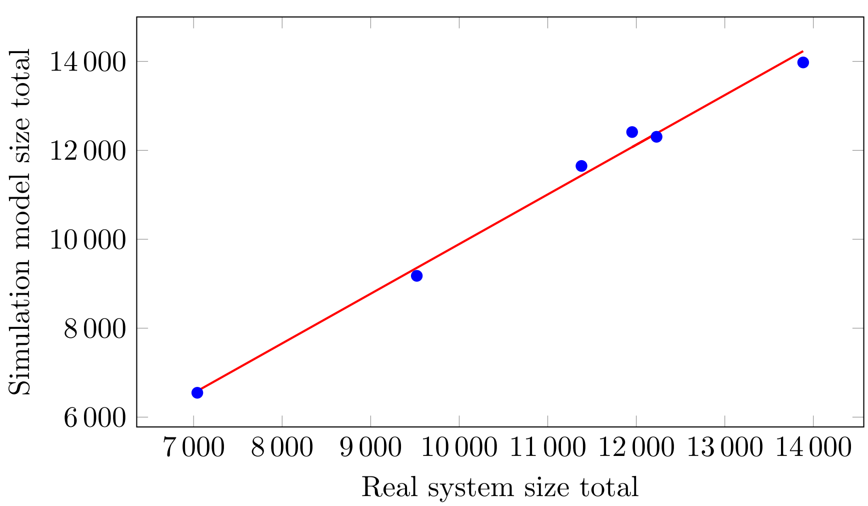

FIGURE 4.16: *Scatter plot of correlation between total real system size sales and total simulated size sales for Subclass W2. The simulation was performed using* $\gamma = 1$.

CHAPTER 5

Results

Contents

The main objective of this study is to analyse the effect of dynamically adjusting size profiles. In this chapter, results obtained from simulating sales with dynamic size profile adjustments are presented using the simulation model described in Chapter 3. The proposed simulation model, using a weighting parameter, $\gamma = 1$, generates total sales for each subclass as expected. Chapter 4 verifies and validates that the simulation model generates a sufficiently accurate representation of the Retailer's real system under the condition of static size profiles. This chapter presents the effect of dynamically adjusting size profiles throughout the season, for each subclass considered in this study.

The balance between historic and current sales data is pivotal to the success of size profile adjustments. The effect of size profile adjustments are analysed for each of the four subclasses considered in this study. Analysis of dynamic size profile adjustments for summer Subclasses S1 and S2 are available in §5.1 and §5.2, respectively. Followed by an analysis of winter Subclass W1 in §5.3, and Subclass W2 in §5.4. Value variation of γ is experimented with in sensitivity analysis for each subclass, identifying the values of γ that effectively responds to changing customer demand by increasing sales. Statistical analysis of the difference in average sales generated by the static and dynamic system, follows the sensitivity analysis. A weighting parameter of 1 is used in the static system to reflect the Retailer's real system (validated proposed simulation model). The dynamic system adjusts size profiles to reflect the current sales performance at stores throughout the season. The chapter ends with a summary of results, available in §5.5.

5.1 Subclass S1

Input parameters used for the simulation of sales for Subclass S1 are validated in §4.3.1 to have sufficient reliability in the representation of reality. Using the verified and validated input parameters and $\gamma = 1$, the proposed simulation model which facilitates the dynamic adjustment of size profiles is validated in §4.4.1 to generate total sales that are as expected and sufficiently accurate compared to the real system. Use of $\gamma = 1$ ensures size profiles (as determined by the Retailer) remain static throughout the season. The validated simulation model is used to generate sales for Subclass S1 where size profiles dynamically adjust throughout the season. Analysis of the effect on total sales observed from the dynamic adjustment of size profiles is reported on in this section for Subclass S1.

Subclass S1 has 1 279 stores, each store carries 6 sizes of the product (Ladies fancy sandals) from UK size 3 to UK size 8. Throughout the season, which comprises of exactly 26 weeks ranging from the 02 August 2014 – 24 January 2015, thirteen styles of the subclass arrive from factories and are allocated to stores according to the size-mix allocation. One of the thirteen styles has no allocation data available and the size-mix allocation is unable to finalise inflow decisions. It is assumed that finalised inflow decisions (as would have been calculated by the size-mix allocation) are equal to the Retailer's actual inflows (as provided). Therefore, twelve opportunities for dynamic size profile adjustments remain for Subclass S1.

This section includes sensitivity analysis of size profile adjustments for various values of γ in §5.1.1, and statistical analysis of the difference between static and dynamic system output is presented in §5.1.2. The effect of dynamic size profile adjustments are analysed weekly on a company level (all stores and sizes) followed by an analysis on the effect at stores, for groups of stores.

5.1.1 Sensitivity analysis

Sensitivity analysis is performed to determine an appropriate weighting parameter, γ, value to use in dynamic size profile adjustments for Subclass S1. Ten simulation replications of each varying parameter are performed and the average output is reported on in Table 5.1. The final

Weighting parameter	Simulated total sales (avg)
$\gamma = 0.1$	88 507.2
$\gamma = 0.2$	89 186.7
$\gamma = 0.3$	89 652.2
$\gamma = 0.4$	89 787.5
$\gamma = 0.5$	89 834.7
$\gamma = 0.6$	89 915.8
$\gamma = 0.7$	89 921.5
$\gamma = 0.8$	89 909.8
$\gamma = 0.9$	89 616.6
$\gamma = 1$	89 084.3

TABLE 5.1: *Sensitivity analysis of S1 simulated total average sales for varying γ values, including static size profile sales where $\gamma = 1$.*

row ($\gamma = 1$) indicates the output generated by the simulation model when size profiles remain static throughout the season, as validated.

Figure 5.1 graphically displays the total sales generated by the simulation model for varying values of γ. The final bar represents average total sales generated by the static size profile simulation. In comparison to the static size profile ($\gamma = 1$) total sales, the total sales generated by each varying value of γ have an effect on total sales.

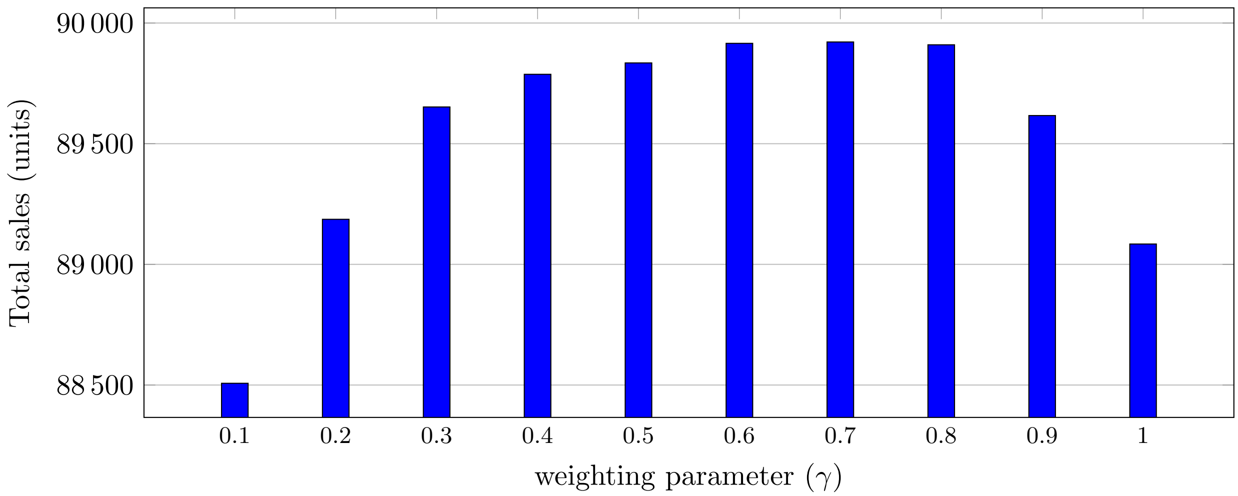

FIGURE 5.1: *Subclass S1 total sales comparison for varying weighting parameter (γ) values, where $\gamma = 1$ reflects static size profile sales.*

Figure 5.2 presents the percentage change in total sales when size profiles are adjusted dynamically throughout the season, compared to sales generated by the static system ($\gamma = 1$). Each bar in Figure 5.2 represents the average percentage change in total sales for each corresponding γ value on the x-axis, compared to the average static size profile total sales ($\gamma = 1$). The best percentage improvement on total sales occurs when $\gamma = 0.7$ is used to dynamically adjust size profiles throughout the season.

Using the weighting parameter $\gamma = 0.7$, total sales increase by 0.94% compared to static system sales, equivalent to an additional 837.2 units of S1 stock that is sold during the season, on average. According to this value of γ, throughout the season a new adjusted size profile is determined which represents 70% of the static/historical size profile (as determined by the Retailer) and 30% of the current size profile (as recorded by the simulation). A store's current size profile represents the cumulative weekly sales to date, recorded in each size at every store until the time of adjustment (the week when stock arrives at the DC from factories). If size profiles adjust dynamically using either $\gamma = 0.6$ or $\gamma = 0.8$, total sales improve by 0.93%, only 0.01% less than when $\gamma = 0.7$ is used.

Dynamically adjusting size profiles using $\gamma = 0.1$ results in a total sales decrease of 0.65%, compared to when no size profile adjustments are performed. The use of this weighting parameter, results in an adjustment of size profiles where 10% of the weight is assigned to the historical size profile, as determined by the Retailer and 90% is assigned to the current sales, as recorded by the simulation model from the start of the season until the time of allocation.

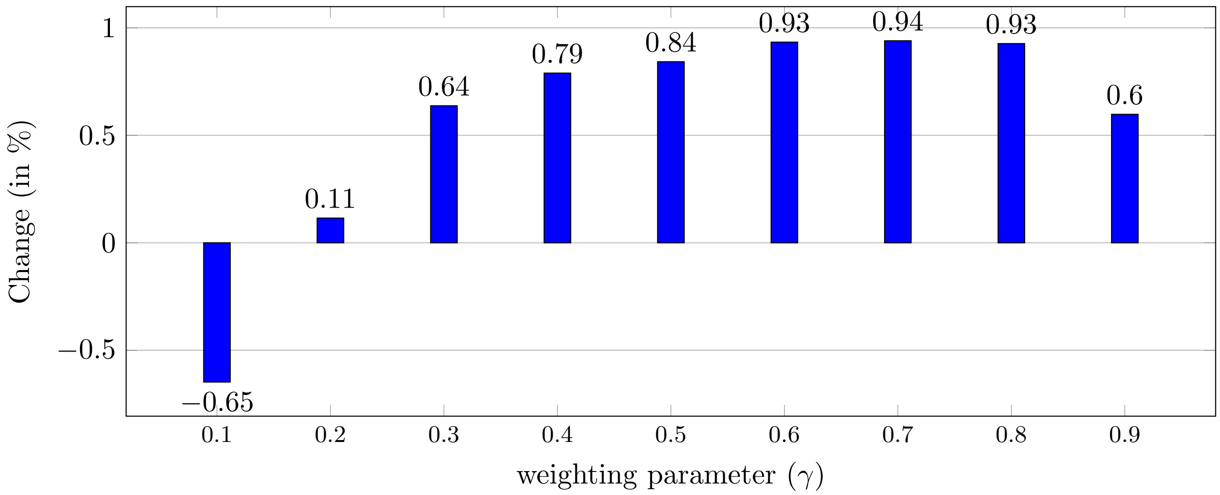

FIGURE 5.2: *Subclass S1 average percentage improvement in total sales for varying values of the weighting parameter (γ), compared to static size profile sales.*

5.1.2 System comparison

Total sales are simulated for the first system (static size profile) and the second system (dynamic size profile adjustments). Each systems sales are simulated ten times. Sensitivity analysis identifies that the largest percentage improvement in total sales for Subclass S1 occurs when size profile adjustments are done using $\gamma = 0.7$, compared to sales generated by the static system ($\gamma = 1$). Statistical analysis on the difference in sales generated by the systems determines whether the change in total sales is due to dynamic size profile adjustments (system design differences), or simply due to random fluctuation inherent in the simulation model [10].

The calculation of confidence intervals assumes the difference in system simulations are normally distributed. Due to the sample size ($n = 10$), Table 5.2 presents two formal tests of normality, obtained via Python 3.6.3 [32] as other formal tests of normality require more data. The null hypothesis for both of these tests states that the difference in total sales are a random sample from a normal distribution, against the alternative hypothesis of non-normality in the data. At a significance level of 0.05, the p-value of each test is larger than the level of significance. Therefore, the null hypothesis is not rejected and it is assumed the difference in sales are normally distributed.

Test	Statistic	p value
Shapiro-Wilk	0.936	0.508
Anderson-Darling	0.313	0.684

TABLE 5.2: *Statistical test of normality for system comparison in Subclass S1.*

Total sales of the second system (dynamic size profiles) are subtracted from the first system (static size profiles). The mean difference between the data series is -837.2 units and the standard deviation is equal to 255.7 units. With 10 data points, the t-distribution has 9 degrees of freedom and consequently, an estimated 95% confidence interval for the difference in means is given by $[-1\,020.12, -654.28]$, where the negative sign is due to the calculation of differences (dynamic system sales, using $\gamma = 0.7$ are subtracted from static system sales, using $\gamma = 1$). Since the whole confidence interval lies below 0, it is concluded that the system with adjusting size profiles, where $\gamma = 0.7$; are performing better than the system with static size profiles.

Sensitivity analysis recorded an increased percentage change in total sales where weighting parameters were within the range $\gamma = 0.2$–0.9. The difference in system output between each weighting parameter and the static system are normally distributed for all values within the range. However, an estimated 95% confidence interval of the difference in means for $\gamma = 0.2$ is given by $[-337.83, 133.03]$. As this confidence interval lies on either side of 0, the test of statistical difference is inconclusive for $\gamma = 0.2$.

The range of weighting parameters which result in statistically different sales due to dynamically adjusting size profiles, include values within the range $\gamma = 0.3$–0.9, compared to the static system. Concluding that a positive percentage change in total sales are recorded for Subclass S1 when size profiles dynamically adjust for weighting parameters within the range $\gamma = 0.3$–0.9 and the largest average percentage improvement is recorded when $\gamma = 0.7$.

Weekly analysis

Having established that the observed difference in total sales are due to differences in system design, the effect dynamic size profile adjustments have on weekly total sales (for all stores and sizes, all styles are considered together) are analysed in this section.

Figure 5.3 plots weekly sales for both systems comparatively. Blue bars indicate average weekly sales when size profiles remain static and red bars indicate average weekly sales when size profiles adjust dynamically, using $\gamma = 0.7$. When styles arrive at the DC for allocation to stores, size profiles are dynamically adjusted. Styles arrive in weeks 2, 6, 8, 10, 15, 18, 20, 23 and 24 meaning there are eight opportunities for size profile adjustments to occur. The first style in week 2 is sent according to the historic size profile as no sales in this season have been recorded, thus no size profile adjustments are possible.

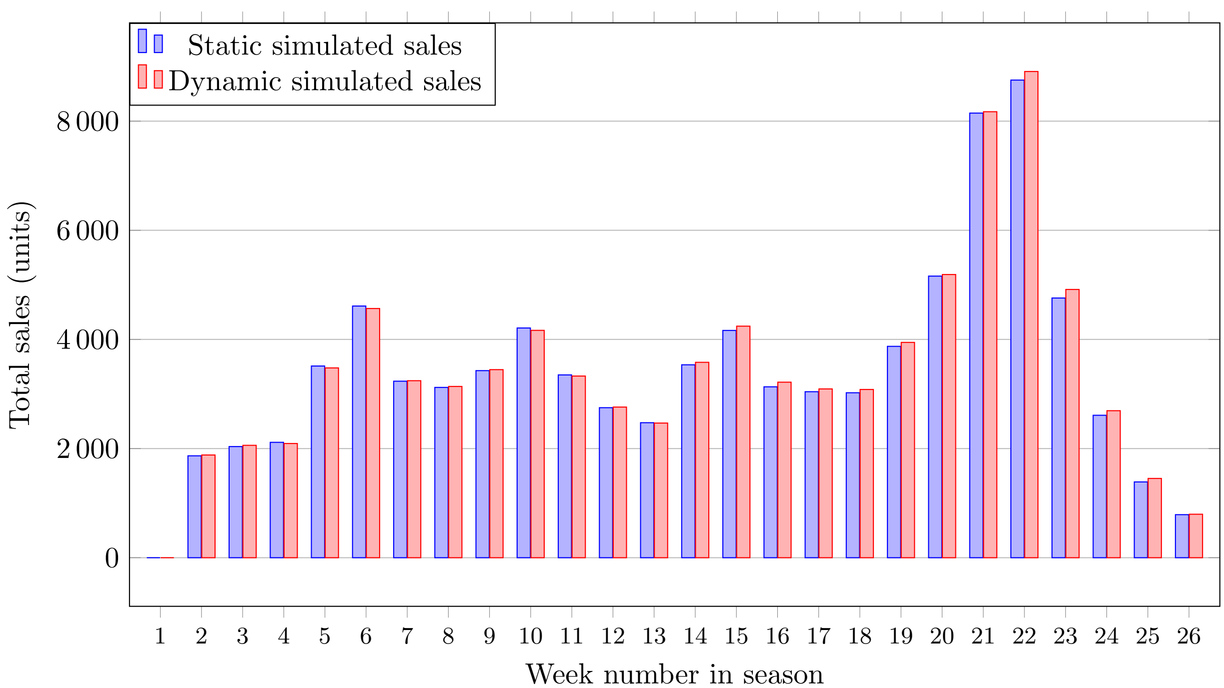

FIGURE 5.3: *Subclass S1 comparison of average weekly simulated sales for the two systems under consideration.*

Figure 5.4 provides a clear visualisation of the weekly change in total sales experienced when dynamic size profile adjustments are performed. Weekly sales generated when size profiles remain static are subtracted from the recorded weekly sales generated when size profiles dynamically adjust ($\gamma = 0.7$), presenting the weekly change in sales when size profiles dynamically adjust throughout the season. Of the 25 weeks in the season where sales occur (excluding week 1 as there are no inflows, thus no sales), 19 weeks record an increase in sales due to dynamic size profile adjustments.

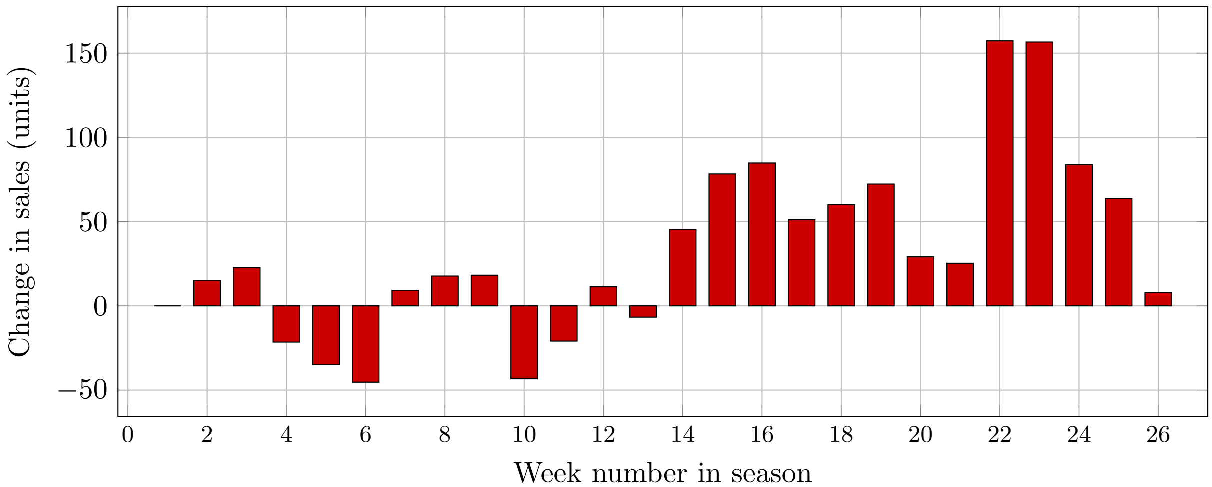

FIGURE 5.4: *Weekly difference in total sales of Subclass S1 generated using $\gamma = 0.7$ compared to total sales generated using a static size profile.*

The first opportunity for size profiles to dynamically adjust is in week 6, as week 2 receives stock according to the static size profile (no sales recorded to date). Sales in weeks 4, 5 and 6 decrease on average by 21.5, 34.8 and 45.3 units, respectively, compared to static size profile sales. Stock arrives at the DC in weeks 8 and 10, where size profiles dynamically adjust. In weeks 7, 8 and 9 sales increase by a total of 45.1 units compared to static size profiles and weeks 10 and 11, sales decrease again. Sales in week 12 increase slightly followed by a final decrease in total sales, recorded in week 13. From week 14, the system of dynamic size profile adjustments record an increase in total weekly sales until the end of the season.

A possible reason for the initial decrease in sales generated by the dynamically adjusting simulation model using $\gamma = 0.7$, is due to volatility in the initial few weeks of sales, leading to a slight overcompensation in adjustment. However, the remaining five size profile adjustments in weeks 15, 18, 20, 23 and 24 all record increased total sales, indicating that the initial potential volatility in total sales amongst stores has stabilised. In the first few weeks the effect of the adjusted size profiles would also be smaller on sales, because not a lot of current sales data is available to adjust the profiles.

Two performance evaluation metrics which quantify both surplus stock and stock shortage, evaluate weekly performance of both systems under a holistic view of company performance. Surplus stock is quantified by the shipment success ratio (SSR) which represents the fraction of all units of the subclass shipped to stores since the start of the season that have actually been sold to date. The demand cover ratio (DCR) quantifies stock shortage, representing the proportion of demand that was successfully converted into sales since the start of the season to date.

A scatter plot of the performance metrics are presented by way of an evaluation framework, available in Figure 5.5. Blue squares represent weekly performance when size profiles remain static, and red diamonds represent the corresponding weekly performance of dynamically adjusting size profiles using $\gamma = 0.7$. The calculation of each weekly performance metric considers cumulative sales, inflow and demand to date for both systems (independently). As the metrics represent cumulative performance, points move from left to right along the x-axis, and from the bottom, up along the y-axis. An ideal situation is recorded where points are closer to 1 along both axes. A red diamond to the right and above its corresponding blue square implies an improvement in the particular week due to dynamic size profile adjustments.

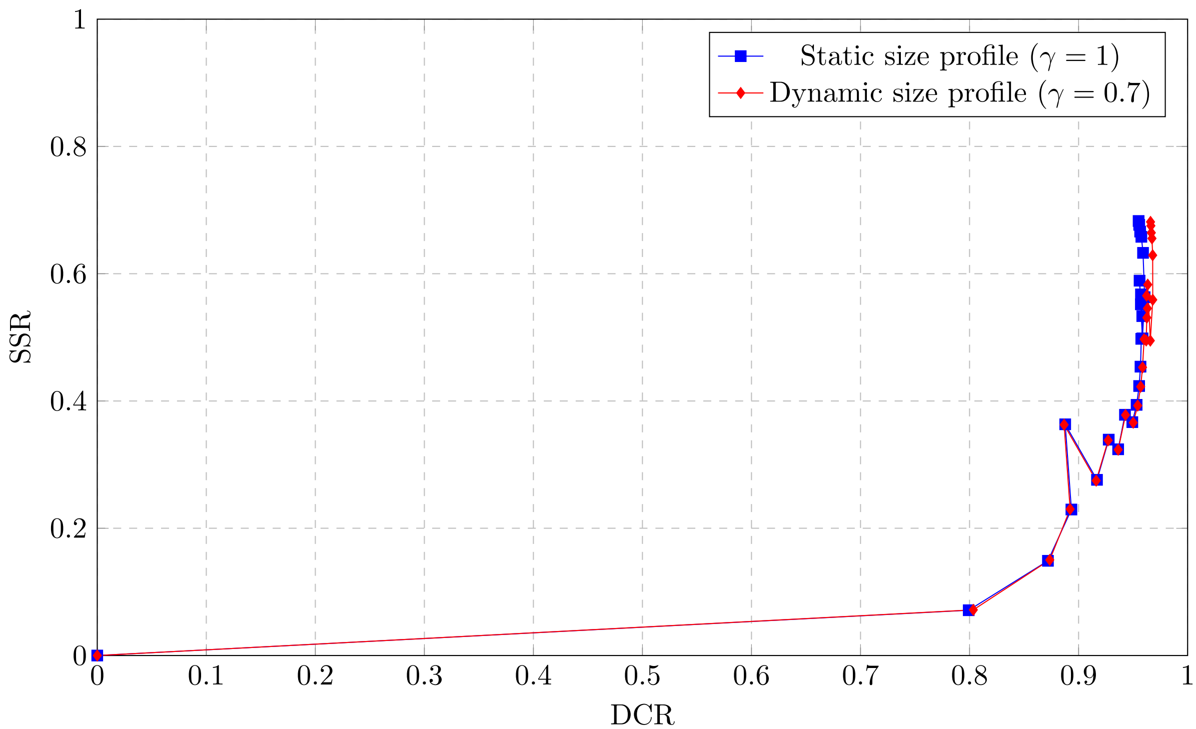

FIGURE 5.5: *Subclass S1 company evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.7$.*

As a whole, Subclass S1 performs better in most weeks when dynamic size profile adjustments occur (red diamonds are closer to 1 along the axes than corresponding blue squares). Points below 0.5 on the y-axis indicate that there is too much inventory everywhere. Meaning, the amount of stock that all stores have received is more than the amount of recorded sales to date. This is understandable and acceptable for the beginning of the season when more stock is usually available in anticipation of future demand. Week 5 has point coordinates at (0.89, 0.36) which is quite high for the start of the season. However, in week 6, when a new style is allocated, the coordinates move down to (0.92, 0.27). Between these two weeks, points increase along the x-axis, indicating the proportion of demand that was able to be converted into sales increased. A decrease along the y-axis between these two weeks is indicative of increased inflows as the proportion of sales given inflow to date has decreased.

Two styles are allocated in week 10 and from then onwards, weekly red diamonds shift further to the right of blue squares, indicating greater demand satisfaction (more demand was able to be converted into sales, meaning more weekly sales were recorded and not more inflow in total, simply from better allocation of sizes amongst stores). The season ends with red diamonds closer to 1 on the x-axis, meaning size profile adjustments satisfied demand to a greater extent, thus fewer stockouts occurred weekly and more stock is sold overall resulting in less left over stock at the end of the season. The dynamic adjustment of size profiles do not influence the total inflow amount and red diamonds are on par with blue squares along the y-axis.

Weekly store analysis

In the weekly analysis of the company (all stores and sizes) performance it was confirmed that using $\gamma = 0.7$ when performing size profile adjustments indeed increased weekly sales throughout the season. In this section, stores are grouped together by their relative total inflow (for Subclass S1, for the season) to analyse whether various groups of stores are affected differently by dynamic size profile adjustments. Stores are sent larger inflow quantities when sales for the season are anticipated to be large, and smaller inflow quantities are sent to stores where sales are anticipated to be small for the season. Analysis of the total inflow for each of the 1 297 stores in Subclass S1, indicate a minimum total inflow received at a store is 16 units and the maximum total inflow a store receives is 482 units, for the season.

Table 5.3 categorises stores into quartiles relative to their inflow quantities. Each category represents 25% of stores in this subclass, the range of inflows for each category are indicated in the third column and the percentage each category contributes to total inflow is presented in the last column. Category 1 and 2 make up 50% of the stores, of which the total inflow contribution sums to 27.33%, indicating that the majority of stores in this subclass are considered small. Stores in Category 3 are considered medium and large stores are grouped into Category 4.

Category	% of stores	Range of inflow (MIN-MAX)	% of total inflow
1	25%	16–55	10.72%
2	25%	56–82	16.61%
3	25%	83–129	25.92%
4	25%	130–482	46.75%

TABLE 5.3: *Subclass S1 categorisation of stores given inflows.*

Analysis of the effect size profile adjustments have at different store sizes are presented in this section. When $\gamma = 0.7$ is used to adjust size profiles, total sales for stores in each category increase. On average total sales for stores in Category 1, 2, 3 and 4 increase by 0.69%, 1.31%, 0.97% and 0.86% respectively, compared to their corresponding static size profile sales. With regards to unit increases, these percentages equate to an additional 54.2, 173, 215.7 and 394.3 units sold in each respective category, on average. Each category's evaluation framework of weekly performance for the static system and the dynamic system, using $\gamma = 0.7$ are presented in Appendix A.1.1.

Considering stores within these categories, the effect of dynamically adjusting size profiles using extreme weighting parameter $\gamma = 0.1$ and $\gamma = 0.9$ values, are analysed. Dynamically adjusting size profiles using $\gamma = 0.1$, result in an average decrease in sales, for stores within each category compared to static size profile sales. Stores in Category 1, 2, 3 and 4 sell on average 2.22%, 1.25%, 0.77% and 0.15% less stock, respectively, over the season when $\gamma = 0.1$ is used, compared to static system total sales.

Category 1 contains 25% of stores in Subclass S1 categorised as receiving the smallest inflow, relative to total inflow and Category 4 contains 25% of Subclass S1 stores categorised as receiving the largest inflow, for the season. To understand the effect experienced at these stores the weekly sales performance is presented by means of an evaluation framework. Within this framework, a point to the right and above its corresponding point is the other system implies an improvement. Figure 5.6 presents the evaluation framework of stores in Category 1 when dynamic size profile adjustments using $\gamma = 0.1$, and Figure 5.7 presents stores in Category 4. The evaluation frameworks for stores in Category 2 and 3, as well as an evaluation framework of the company are available in Appendix A.1.2.

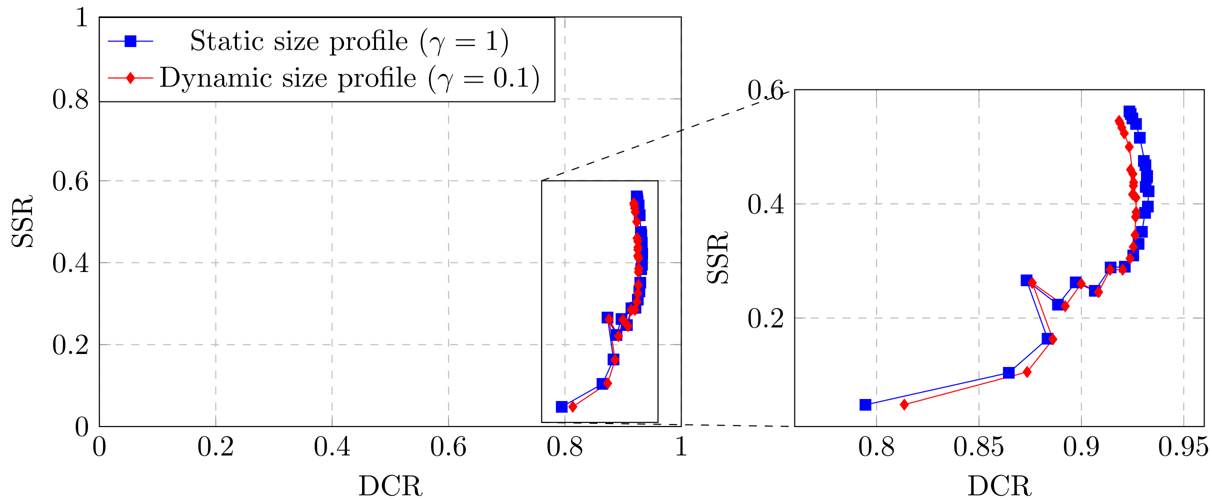

FIGURE 5.6: *Subclass S1 evaluation framework of static system performance and dynamic system performance, using $\gamma = 0.1$. Considering stores in Category 1.*

In Figure 5.6 red diamonds are to the right of corresponding blue squares for weeks 2–8 where dynamic size profile adjustments are performed in weeks 6 and 8. Thereafter, red diamonds shift to the left of corresponding blue squares and the season ends with red points lower than blue points along the y-axis which implies an overall decrease in Category 1 store performance. Smaller stores experience more volatile sales throughout the season and $\gamma = 0.1$, assigns 90% of the adjustment to reflect current sales. It may be inferred that these drastic dynamic adjustments are ill suited for stores within Category 1 (small stores).

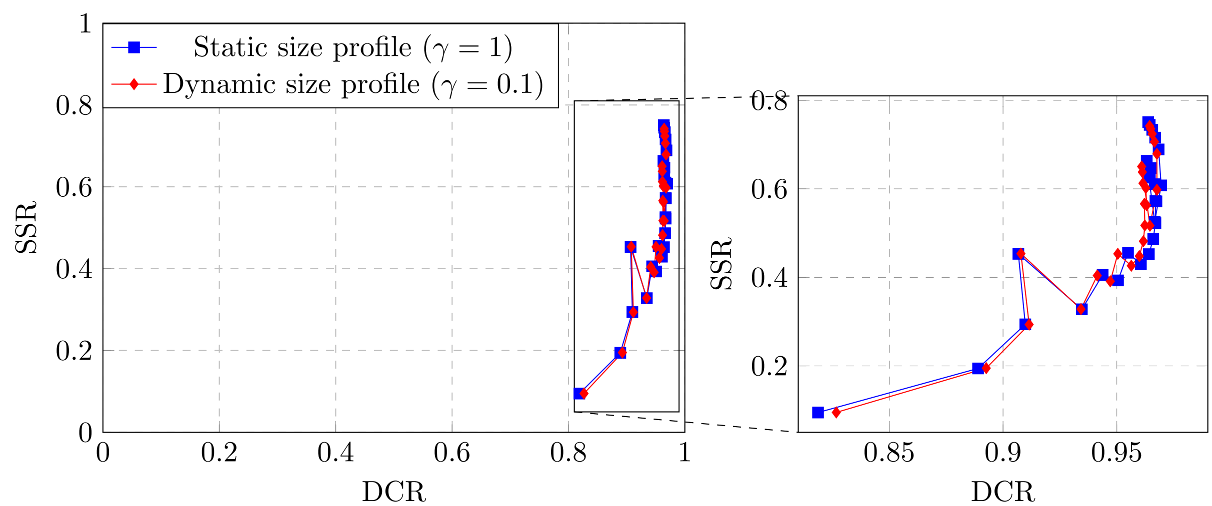

FIGURE 5.7: *Subclass S1 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.1$. Considering stores in Category 4.*

The evaluation framework for stores in Category 4 are presented in Figure 5.7, where the weekly movements between the blue and red points (static system and dynamic system) are less pronounced in comparison to the movement of stores in Category 1 (Figure 5.6). However, the general leftwards shift of red points implies a decrease in demand satisfaction, resulting in more stockouts throughout the season. At the season's end, the two systems are relatively close to one another along the x-axis but red points are slightly below blue points on the y-axis, confirming a decrease in total sales. It is concluded that the use of $\gamma = 0.1$ is not suitable for either categories of stores as total sales decrease in comparison to static system sales.

In contrast, dynamic size profile adjustments using $\gamma = 0.9$ result in an increase in total sales for stores within each of these categories. On average, stores in Category 1, 2, 3 and 4 sell an additional 0.17%, 0.99%, 0.84% and 0.44% stock compared to static size profile sales, respectively. For comparability, weekly performance of stores in Category 1 are presented in Figure 5.8 and stores in Category 4 are presented in Figure 5.9, when dynamic size profile adjustments are done using $\gamma = 0.9$. The evaluation frameworks for stores in Categories 2 and 3 are available in Appendix A.1.3, as well as the effect on a company level.

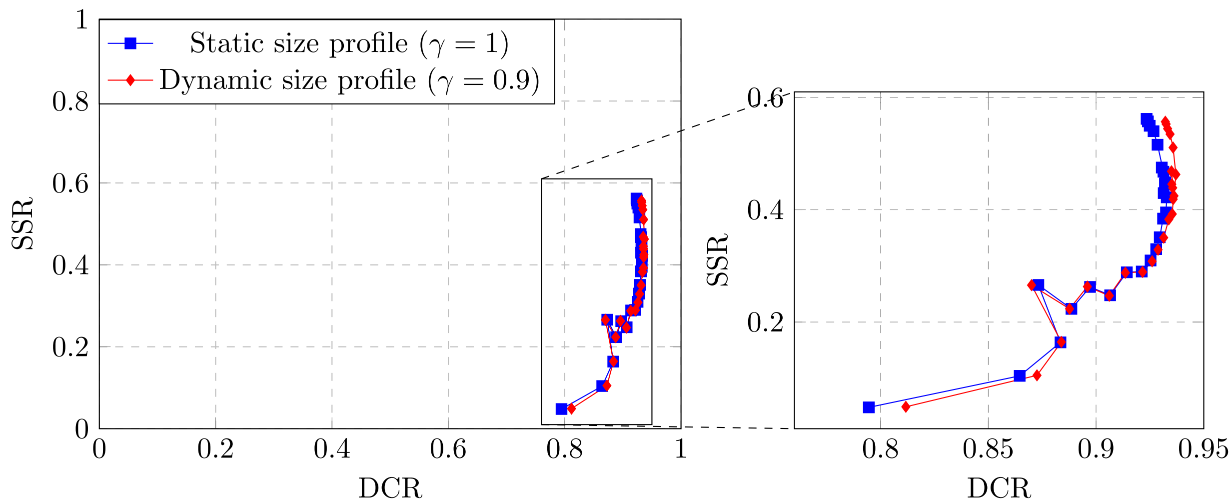

FIGURE 5.8: *Subclass S1 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.9$. Considering stores in Category 1.*

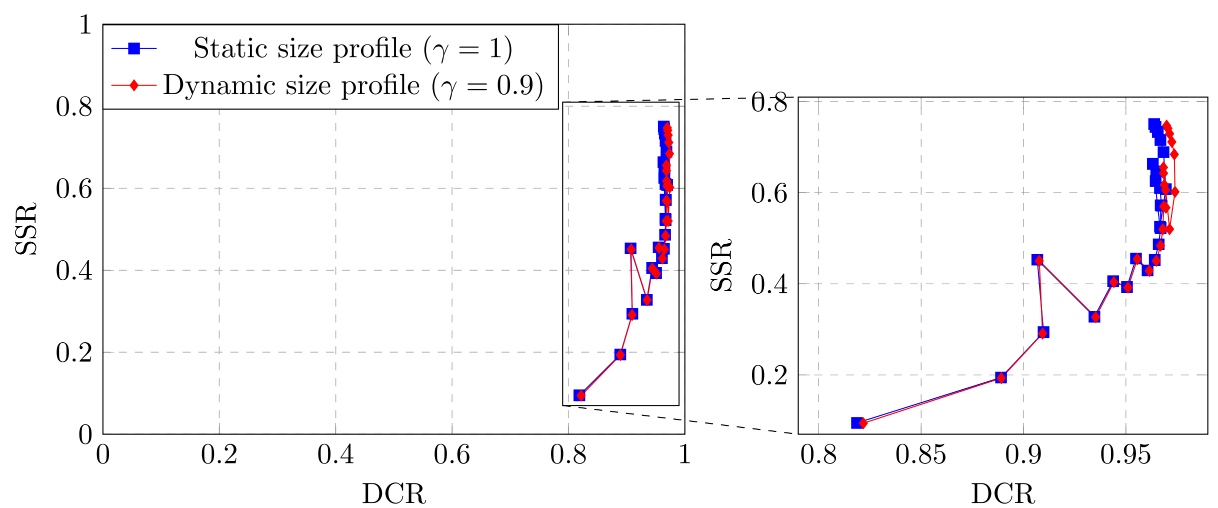

FIGURE 5.9: *Subclass S1 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.9$. Considering stores in Category 4.*

Figure 5.8 indicates increased demand satisfaction weekly for stores within Category 1. In the last week of the season, red points are above the blue points (the fraction of sales given inflows is higher than in the static system), meaning more sales have occurred throughout the season and less surplus stock remains at the end of the season. As stated previously, stores in this category are considered small and often experience volatile sales. Size profile adjustments where 90% of the adjusted size profile comprises of the historical size profile, enforces more stability in size profile adjustments and thus avoid overcompensation.

Figure 5.9 presents the weekly performance of stores in Category 4 which are considered large stores. In week 6 (when the first size profile adjustment occurs), the red point is marginally higher than the corresponding blue point, indicating that the fraction of sales given inflows to date is marginally better than the static system. Thereafter, slight improvements along both axes are reported weekly until the season's end.

In comparison to the evaluation framework for $\gamma = 0.1$, it is clearly visible that $\gamma = 0.9$ results in an overall improvement in total sales compared to static system sales. However, compared to the chosen weighting parameter of $\gamma = 0.7$, the use of $\gamma = 0.9$ is not as significant and it is recommended to dynamically adjust size profiles for Subclass S1 using $\gamma = 0.7$. In conclusion, when the majority of stores in a subclass is small, using a higher weighting parameter value results in more favourable outcomes than using a weighting parameter which is too small, such as $\gamma = 0.1$.

5.2 Subclass S2

This section analyses the effect of dynamically adjusting size profiles for the second summer subclass considered in this study, Subclass S2. This subclass is smaller than S1, containing 969 stores and a total of only three styles for the season. Each store carries five sizes of the product (Mens fancy sandals), ranging from UK size 6 to UK size 10. Sales are simulated for the season, containing exactly 26 weeks and ranging from 02 August 2014 – 24 January 2015. The simulation model logic presented in Chapter 3, is verified and validated for S2 in §4.4.2 for input parameters—validated in §4.3.2—and $\gamma = 1$. Output generated via the simulation model is sufficiently close to the real system and the model is concluded to be valid and reliable for the purpose of this study.

Sensitivity analysis on dynamic size profile adjustments are presented in §5.2.1 and a comparison between static and dynamic size profile generated sales is available in §5.2.2. Thereafter, weekly sales are analysed on firstly, a company level and secondly, on a store size level where extreme weighting parameter values are considered.

5.2.1 Sensitivity analysis

Sensitivity analysis is performed to determine an appropriate weighting parameter value to use in dynamic size profile adjustments for Subclass S2. The magnitude of adjustment is dependent on the γ value.

To ensure reliability in the simulation output, the proposed model is replicated ten times for each value of γ and the average output is reported on in Table 5.4. The final row ($\gamma = 1$) represents output generated by the simulation of static size profiles for Subclass S2. Figure 5.10 presents the total sales values reported as a graphic, enabling a comparison between total sales output generated by the simulation model for various values of γ.

Figure 5.11 provides a visual representation of the change in total sales for each value of γ, against static size profile sales. Each bar indicates the percentage change in total sales for the values of γ. When size profile adjustments are performed using $\gamma = 0.1$ or $\gamma = 0.2$, total sales

Weighting parameter	Simulated sales (avg)
$\gamma = 0.1$	12 503
$\gamma = 0.2$	12 640.9
$\gamma = 0.3$	12 751.4
$\gamma = 0.4$	12 834.4
$\gamma = 0.5$	12 896.1
$\gamma = 0.6$	12 900.6
$\gamma = 0.7$	12 948
$\gamma = 0.8$	12 963.8
$\gamma = 0.9$	12 824.6
$\gamma = 1$	12 688.4

TABLE 5.4: *Sensitivity analysis of S2 simulated total average sales for varying γ values, including static size profile sales where $\gamma = 1$.*

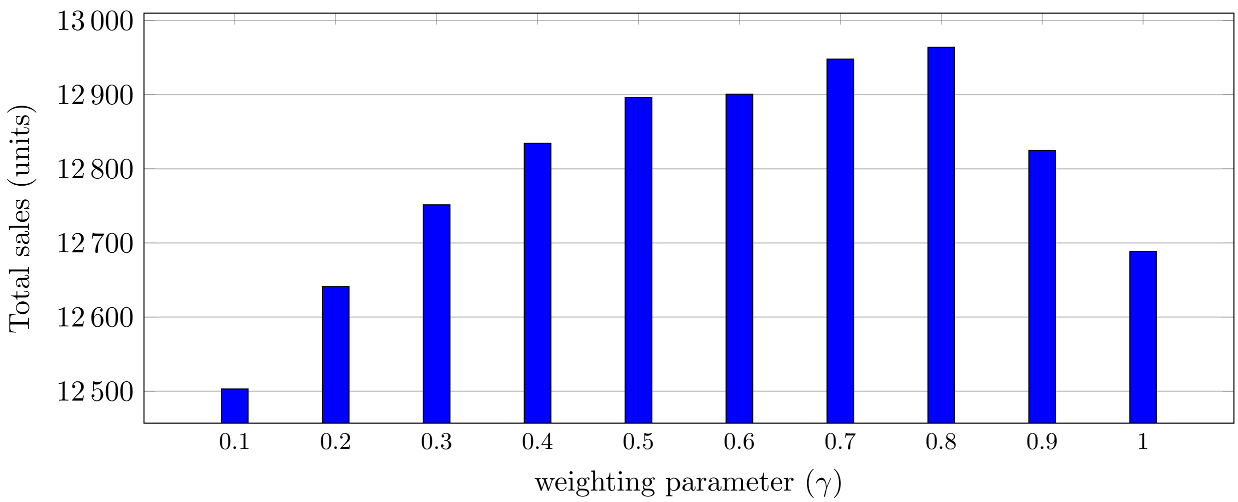

FIGURE 5.10: *Subclass S2 total sales comparison for varying weighting parameter (γ) values, where $\gamma = 1$ reflects static size profile sales.*

decrease by 1.46% and 0.37%, respectively, compared to static size profile sales ($\gamma = 1$). Size profile adjustments using $\gamma = 0.3$ or greater record a percentage improvement in total sales for the season, compared to static size profile sales.

Subclass S2 has a limited number of opportunities for size profile adjustments, as only three styles arrive throughout the season. As stated previously, the first style received by a store is sent according to the size profile determined by the Retailer. No sales have been recorded thus far by the simulation model therefore, no size profile adjustments can occur. Once sales have been recorded in the current season, size profile adjustments may be calculated, limiting the opportunity for size profile adjustments in Subclass S2 to a maximum of two styles, arriving in two weeks throughout the season.

Subclass S2 receives a total of 18 938 units of stock inflow for the season, which is relatively small. It is understood that the quantity of potential sales are a function of stock inflow. Small inflow quantities are indicative of small sales within stores, which are often volatile. As identified in Subclass S1, low values of γ are less favourable when there is potential for volatility in sales because size profile adjustments reflect a weighted ratio between historic and current sales.

The percentage change in total sales compared to the static system sales, for each varying value

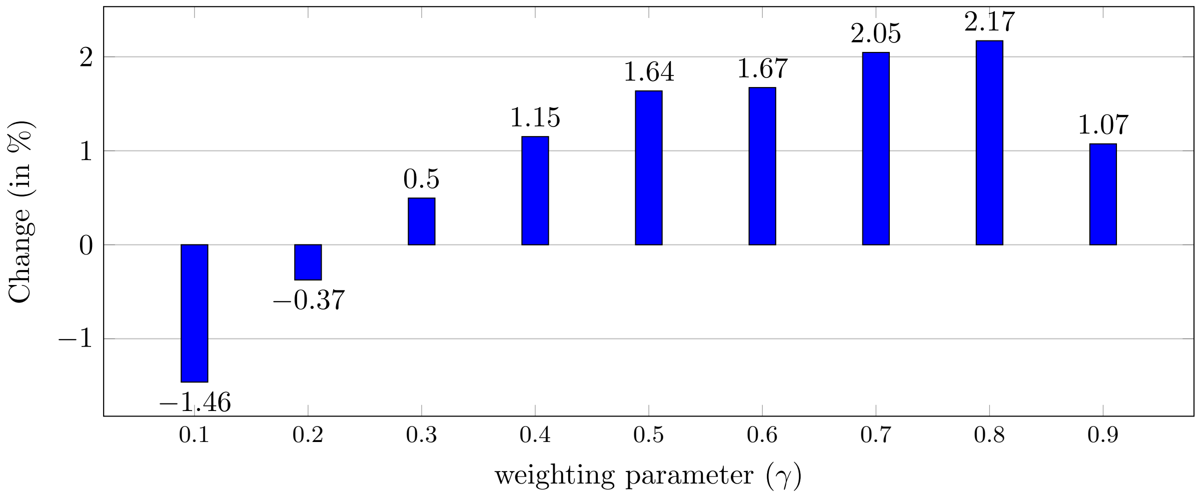

FIGURE 5.11: *Subclass S2 average percentage change in total sales for varying values of γ, compared to static size profile total sales.*

of the weighting parameter is presented in Figure 5.11. When dynamic size profile adjustments are performed using $\gamma = 0.8$, total sales increase on average by 2.17%, compared to static size profile sales. This percentage improvement equates to an additional 275.4 units sold on average, in Subclass S2 over the season.

The use of a larger weighting parameter ($\gamma = 0.8$) makes sense for Subclass S2 due to the relatively small size of the subclass (in terms of inflows and thus sales). For the parameter $\gamma = 0.8$, a resulting balance between historic and current size profiles generates a new size profile which comprises of 80% historical sales and 20% current sales.

5.2.2 System comparison

In this section, statistical analysis is performed to determine whether observed differences between static size profile sales (system 1) and dynamic size profile sales (system 2), where $\gamma = 0.8$ is used; are due to (a) differences in design or simply (b) random fluctuation inherent in the models.

Each system has ten simulation replication output values for total sales. It is assumed the difference in system simulations are normally distributed for the confidence interval to be determined. Due to the small sample size ($n = 10$) only two formal tests of normality are conducted, where the null hypothesis for both tests states that the difference in total sales are a random sample from a normal distribution, against the alternative hypothesis of non-normality. Results of the two tests of normality are obtained via Python 3.6.3 [32] and presented in Table 5.5. At a significance level of 0.05, the p-value of each test is greater than the level of significance. Therefore, the null hypothesis is not rejected for both tests and it is assumed that the difference in total sales are normally distributed.

Test	Statistic	p value
Shapiro-Wilk	0.976	0.942
Anderson-Darling	0.206	0.684

TABLE 5.5: *Statistical test for normality of system comparisons on Subclass S2.*

The dynamic system sales are subtracted from static system sales and the mean difference between the two systems is -275.4 units, with a standard deviation of 164.4 units. The t-distribution has 9 degrees of freedom and at a significance level of $\alpha = 0.05$, an estimated 95% confidence interval for the difference in systems is given by $[-393.01, -157.79]$, where the negative sign is due to the calculation of differences. Given the confidence interval, it is concluded that the mean output from system 1 (static) differs from system 2 (dynamic) and the system with adjusting size profiles where $\gamma = 0.8$ perform better than the system with static size profiles (indicated by the negative sign in the confidence interval).

Sensitivity analysis results, presented in Figure 5.11 indicate a range of values which result in a positive percentage change in total sales, compared to static system sales. Weighting parameter values within the range $\gamma = 0.3$–0.9, are reported to increase sales on average when applied to size profile adjustments. Statistical analysis on the range of values conclude the difference in sales (static system less dynamic system) are normally distributed for each value in the range. However, conducting system comparison tests for the mean difference in sales generated using $\gamma = 0.3$ against static system sales, lead to inconclusive results due to the estimated 95% confidence interval given by $[-153.79, 27.79]$, which lies on either size of zero.

The weighting parameter values that generate statistically different sales, outperforming the static system are within the range $\gamma = 0.4$–0.9. Data are normally distributed and the estimated 95% confidence intervals for values in this range confirm an increase in total sales is due to dynamic size profile adjustments.

Weekly analysis

Sales are simulated weekly and size profile adjustments occur when styles, arriving at the DC from factories, must be allocated to stores. Subclass S2 receives fresh style stock from manufacturers in weeks 4, 9 and 18. As stated previously, the first style sent to a store within the season has no size profile adjustments as there are no current sales on which to adjust. Therefore, only in week 9 and 18 may size profiles dynamically adjust.

Figure 5.12 plots total weekly sales for Subclass S2, averaged over ten simulation replications of firstly, static size profile sales in blue and secondly, dynamic size profile adjustment sales, where $\gamma = 0.8$ is used; in red. For clarity of movement between systems, Figure 5.13 presents the unit change in weekly sales, where weekly static sales, generated using $\gamma = 1$; are subtracted from dynamic size profile adjustment generated sales, using $\gamma = 0.8$. These two figures present the same results, in different ways thus enriching the visual interpretation of weekly sales movement between systems.

In Figure 5.12 the season's overall pattern of sales indicates two major peaks, the first in week 10 and the second in week 22. Total sales in both of these peaks are higher when size profiles dynamically adjustment using $\gamma = 0.8$, than when size profiles remain static. Figure 5.13 indicates that on average (after 10 simulation replications), total sales in week 10 increases by 13.7 units and by 64.5 units in week 22. Of the 23 weeks where sales are recorded, total sales increase in 17 weeks for Subclass S2 when dynamic size profile adjustments are performed.

From these figures, it is clear that before week 4, no inflows arrive in any stores as total sales for both systems (static and dynamic) are zero. The first potential size profile adjustment is in week 9. Sales increases by 10.7 units, on average. From week 9–16, dynamic size profile adjustments record an average increase in total weekly sales. A slight decrease of 4.7 units, on average, is observed in week 17. The second style arrives in week 18 and when size profiles are dynamically adjusted, total sales increase by an average of 9 units. For the remainder of the

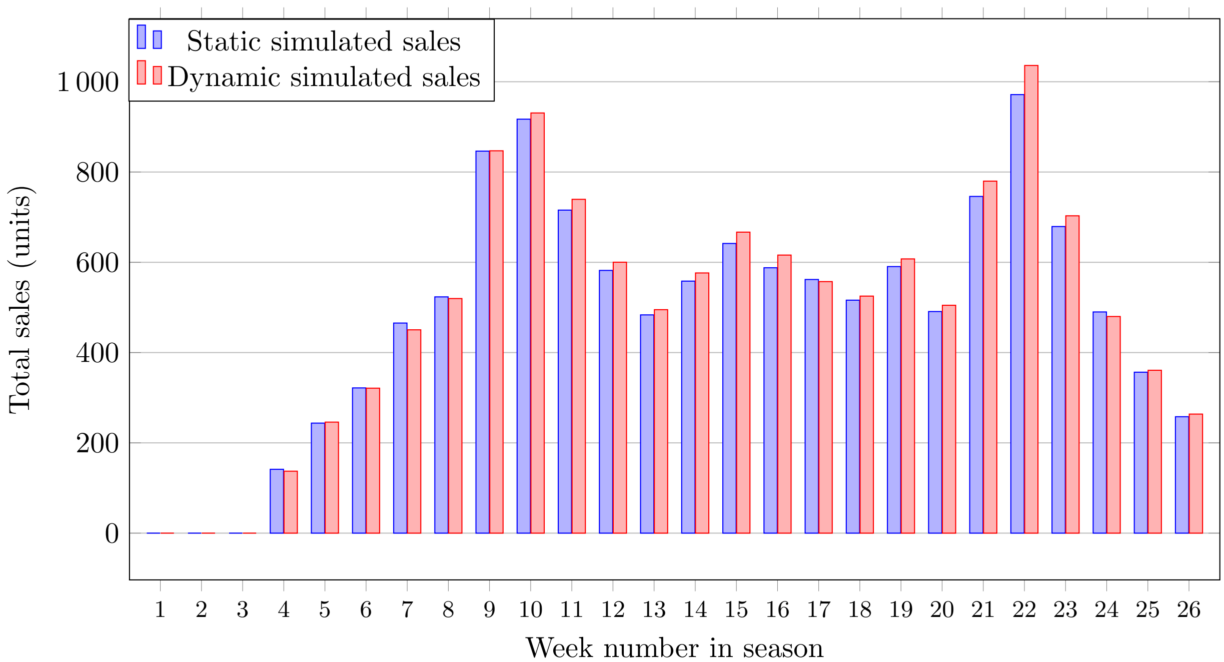

FIGURE 5.12: *Subclass S2 comparison of weekly total sales for the two systems considered, static ($\gamma = 1$) and dynamic ($\gamma = 0.8$).*

season, dynamically adjusting size profiles using $\gamma = 0.8$ increases weekly sales, apart from week 24. The largest increase in total sales occurs in week 22 where an additional 64.5 units are sold, on average, compared to static size profile sales.

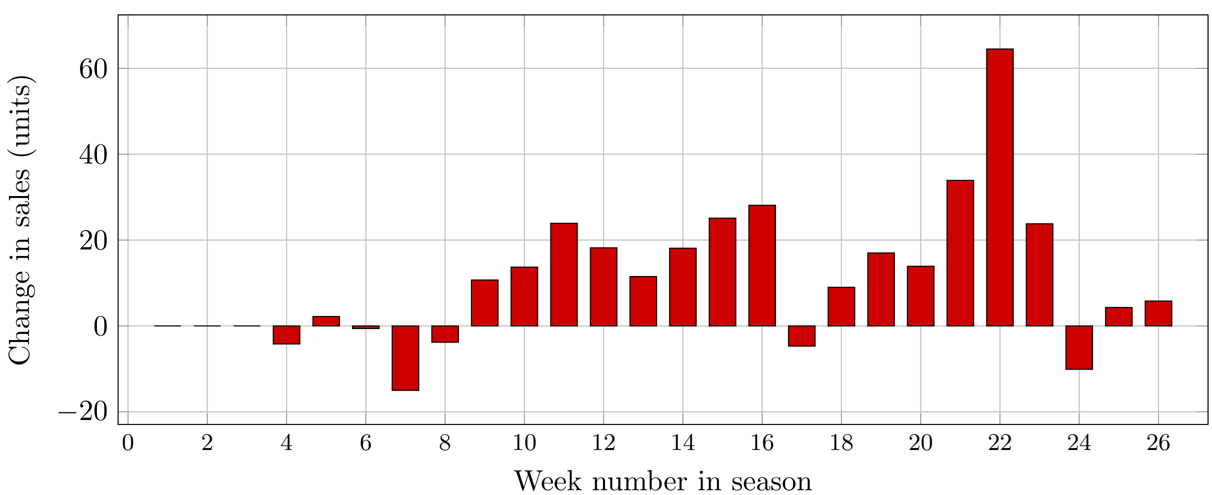

FIGURE 5.13: *Weekly difference in total sales of Subclass S2 generated using $\gamma = 0.8$ compared to total sales generated using a static size profile ($\gamma = 1$).*

Sales are only recorded by the simulation model if (a) there is demand for a unit at a specific store and (b) if there is stock availability. Two performance measures, namely SSR and DCR, are used to quantify the relationship between firstly, sales given inflows (SSR) and secondly, sales given demand (DCR). An evaluation framework in Figure 5.14 presents the resulting weekly relationship between the two systems (static and dynamic, where $\gamma = 0.8$), on a company level (all stores and sizes). Blue squares indicate weekly performance obtained when size profiles remain static. The corresponding red diamonds indicate the effect of dynamically adjusting size profiles using $\gamma = 0.8$. The coordinates of weekly points indicate cumulative sales given firstly demand, on the x-axis and secondly inflows, along the y-axis. As these are cumulative calculations, points move from left to right and values closer to 1, on both axes are favourable.

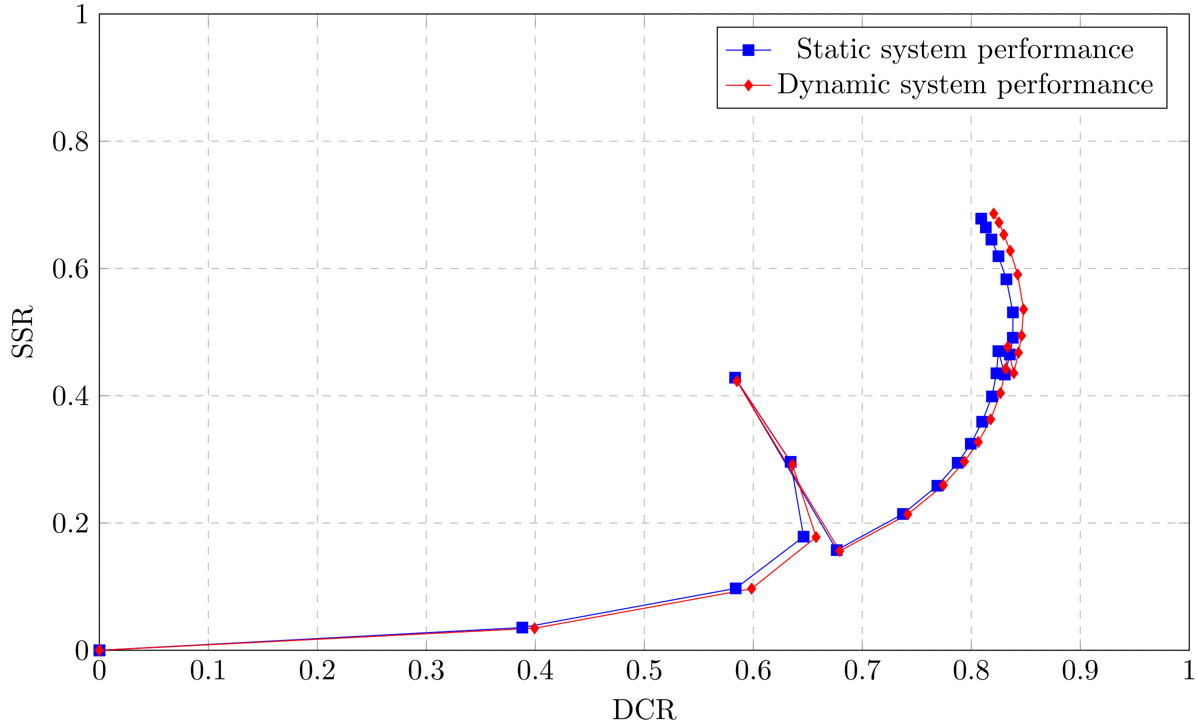

FIGURE 5.14: *Subclass S2 company evaluation framework of static ($\gamma = 1$) system performance and dynamic system performance, where $\gamma = 0.8$.*

The first three weeks have no inflows or sales and therefore, the first point (for both systems) is recorded in week 4 at the coordinates (0.3993; 0.0347) where demand satisfaction from the dynamic system (red diamond) is recorded slightly to the right of the static system (blue square) which is due to random variation in demand as no size profile adjustments have been performed. Weeks 7 and 8 record a sharp upwards move for both systems, the dynamic system (red diamonds) lie slightly below static system performance (blue squares). The upwards move along the y-axis is an indication that available stock is diminishing, as a result cumulative sales relative to cumulative inflow is large.

The following week—week 9—receives new style stock, this is also the first opportunity for size profiles to adjust. The new stock inflow results in a movement down the y-axis for both systems (simply indicating that stock arrived). From week 10 onwards red diamonds (dynamic system) are to the right of corresponding blue squares (static system). The rightwards shift indicates that the proportion of demand able to convert into sales is higher when dynamic size profile adjustments are performed. Meaning, the new size profile is a better reflection of changing customer demand and improve the partitioning of fixed company stock in week 9. This rightwards movement is evident and increasing in magnitude after the second style allocation in week 18, weekly red diamonds diverge to the right of corresponding blue squares until season's end.

Weekly store analysis

Considering the total inflow sent throughout the season to all 969 stores that receive stock of Subclass S2, 46 stores receive 10 units of stock for the season and 341 stores receive a total of 11 units for the season. A further 293 stores receive between 12 and 21 units of inflow for the season and of the remaining 289 stores, 75% receive a total inflow between 22 and 39 units for the season. The maximum inflow received by a store is 285 units (store # 120). Concluding the majority (95% of stores) in this subclass are small and only 5% have medium to large inflows.

Category	% of stores	Range of inflow (MIN-MAX)	% of total inflow
1	4.75%	10–10	2.43%
2	35.19%	11–11	19.81%
3	30.24%	12–21	22.40%
4	29.82%	22–285	55.36%

TABLE 5.6: *Subclass S2 categorisation of stores given inflows.*

Stores are categorised in Table 5.6 according to inflow quartiles. Considering the effect of dynamic size profile adjustment where $\gamma = 0.8$ is used, on average total sales for stores in Category 1 decrease by 0.62% while stores in Category 2, 3 and 4 increase. On average, total sales for stores in Category 2, 3 and 4 increase by 0.93%, 0.76% and 3.07% for the season, respectively. Each category's evaluation framework of weekly performance for the static and dynamic system, where size profiles adjust using $\gamma = 0.8$, are available in Appendix A.2.1.

Subclass S2 receives only three styles throughout the season and small stores in Category 1 and 2, receive just one style allocation. Meaning size profiles remain static for these stores as there is no current sales data available to adjust on. Therefore, any difference in total sales for these stores is due to random variation and only stores in Category 3 and 4 are analysed further in this section. Medium and large stores in Category 3 and 4 receive between two and three style allocations throughout the season. The effect $\gamma = 0.1$ and $\gamma = 0.9$ have on total sales for stores within these categories are analysed.

Total sales for stores in Category 3 and 4 decrease on average by 2.83% and 0.95%, respectively, when dynamic size profile adjustments are done using $\gamma = 0.1$. Figure 5.15 and Figure 5.16 present evaluation frameworks of the effect experienced weekly at stores in Category 3 and 4, respectively. An evaluation framework of the effect on total sales on a company level, as well as for stores in Category 1 and 2 are available in Appendix A.2.2.

Considering both of these Figures 5.15 and 5.16, dynamic adjustment of size profiles using $\gamma = 0.1$ result in a weekly decrease of demand satisfaction (red diamonds to the left of blue squares) from the 14th week. At the end of the season, red diamonds are also slightly below corresponding blue squares indicating a decrease in units sold relative to inflows, resulting in higher surplus stock. It is concluded that Subclass S2 should not use $\gamma = 0.1$ to dynamically adjust size profiles as sales are too volatile and there are not enough style inflows to correct for the initial overcompensation which is as a result of the extreme weighting parameter.

Considering the effect of dynamic adjustments using $\gamma = 0.9$, stores in Category 3 decrease on average by 0.42% which equates to a loss of 10.9 units over the season, and Category 4 records an average increase in total sales of 1.80% or 139.4 units for the season. Figure 5.17 and Figure 5.18 present the evaluation frameworks for Category 3 and 4, respectively. The company evaluation framework indicating the effect of dynamic size profile adjustments, using $\gamma = 0.9$ is presented in Appendix A.2.3, along with evaluation frameworks for Category 1 and 2.

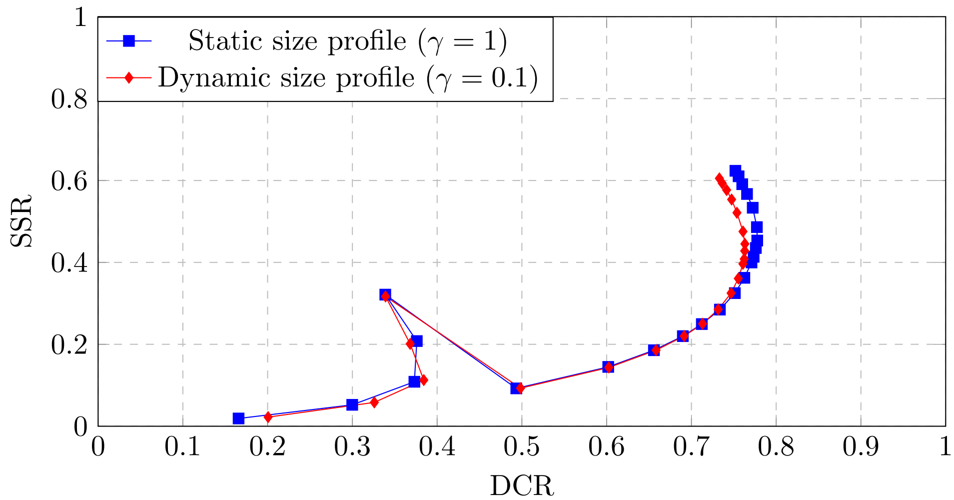

FIGURE 5.15: *Subclass S2 evaluation framework of static system performance and dynamic system performance, where* $\gamma = 0.1$*. Considering stores in Category 3.*

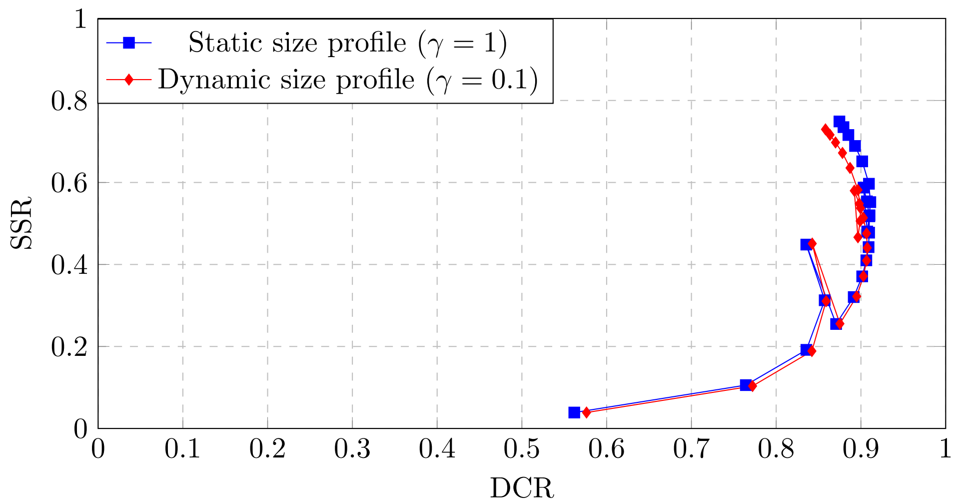

FIGURE 5.16: *Subclass S2 evaluation framework of static system performance and dynamic system performance, where* $\gamma = 0.1$*. Considering stores in Category 4.*

There is no pronounced difference in Category 3's evaluation framework presented in Figure 5.17. Apart from the start of the season where a slight rightward shift of red diamonds are observed in weeks 5 and 6, all remaining weeks red diamonds are relatively on top of corresponding blue squares. Meaning that the use of $\gamma = 0.9$, has no significant impact on total sales for stores in Category 3.

Stores in Category 4 (Figure 5.18) do, however, record an improvement in demand satisfaction for a majority of the weeks during the season (red diamonds to the right of corresponding blue squares). Throughout the season and at season's end, red diamonds are on par with blue squares along the y-axis. Meaning that sales relative to inflow experience no change from the dynamic adjustment of size profiles. Using $\gamma = 0.9$ only slightly influences stock allocation throughout the season, with 90% of the adjustment assigned to historic sales and only 10% to the current sales.

In conclusion, it is not recommended to use either values of the weighting parameter analysed,

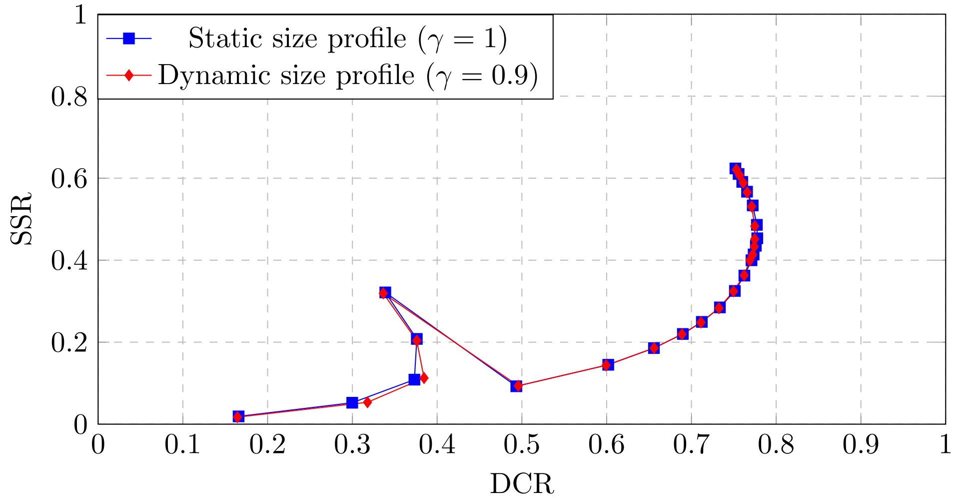

FIGURE 5.17: *Subclass S2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.9$. Considering stores in Category 3.*

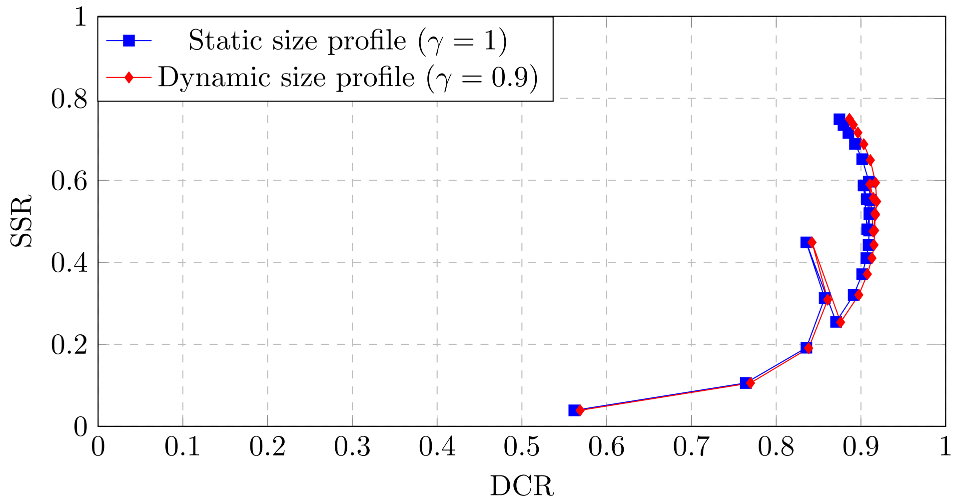

FIGURE 5.18: *Subclass S2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.9$. Considering stores in Category 4.*

$\gamma = 0.1$ or $\gamma = 0.9$, to dynamically adjust size profiles for Subclass S2. In comparison to the use of $\gamma = 0.8$, total sales decrease overall when dynamic adjustments are done using a parameter $\gamma = 0.1$. The decrease in total sales is expected due to the number of small stores in this subclass and the infrequency of style inflows planned for the subclass. Furthermore, it is not recommended to use $\gamma = 0.9$ either as the slight increase in total sales is inferior to the improvement in total sales generated when $\gamma = 0.8$ is used to dynamically adjust size profiles for Subclass S2.

5.3 Subclass W1

The proposed simulation model which facilitates dynamic adjustment of size profiles is validated in §4.4.3 to generate sales that are sufficiently similar to the real system sales when $\gamma = 1$ is used. The use of $\gamma = 1$ ensures size profiles do not dynamically adjust and remain as the Retailer determined.

This section presents results obtained from the simulation of sales for Subclass W1 that incorporates current sales data into the allocation process of the validated simulation model. The season consists of exactly 26 weeks and ranges from 08 February 2014 – 02 August 2014. There are 1 273 stores that receive stock of Subclass W1 for the season and each store carries six sizes, in UK size 3–8 of the product (Teenage girls fancy slippers). Throughout the season, eleven styles are planned to arrive at the DC from factories for Subclass W1. Allocation data is available for nine of the eleven styles sent to stores, meaning dynamic size profile adjustments are performed a possible 9 times, if stores meet the adjustment requirements.

In this section, results from sensitivity analysis of weighting parameter value variation are reported on in §5.3.1. A comparison between total sales generated via the of static system ($\gamma = 1$) and the dynamic system is presented in §5.3.2. Thereafter, weekly analysis on the effect of size profile adjustments is presented firstly, on a company level and secondly, on a store level to analyse the effect of dynamic size profile adjustments on various stores.

5.3.1 Sensitivity analysis

Sensitivity analysis is performed to determine an appropriate value of γ to use in the adjustment of size profiles for Subclass W1. The weighting parameter values tested range from 0.1–0.9, in 0.1 increments. Sales for the season are simulated weekly, size profiles adjust dynamically throughout the season when stock arrives at the DC using most recent sales data available. In the calculation of a new size profile, γ is associated with the historical size profile—as determined by the Retailer using historical sales data—and $1 - \gamma$ is associated with the current spread of sales across sizes (current size profile) at a store.

To ensure reliability in the reported results, the simulation model is replicated 10 times for each varying value of γ. The average sales are reported on in Table 5.7, where the final row presents total sales simulated by the validated static simulation ($\gamma = 1$) where no size profile adjustments occur.

Weighting parameter	Simulated sales (avg)
$\gamma = 0.1$	154 782
$\gamma = 0.2$	155 223.2
$\gamma = 0.3$	155 498.4
$\gamma = 0.4$	155 650.3
$\gamma = 0.5$	155 756.1
$\gamma = 0.6$	155 655.1
$\gamma = 0.7$	155 548.1
$\gamma = 0.8$	155 313.1
$\gamma = 0.9$	155 020.7
$\gamma = 1$	154 068.7

TABLE 5.7: *Sensitivity analysis of W1 simulated total average sales for varying γ values, including static size profile sales where $\gamma = 1$.*

Figure 5.19 presents the average total sales generated by the simulation model for each varying value of γ. The static size profile average total sales are presented in the final column, $\gamma = 1$. It is evident from the bar plot of total sales that each value of the weighting parameter in the dynamic adjustment of size profiles result in higher total sales, compared to sales generated when size profiles remain static.

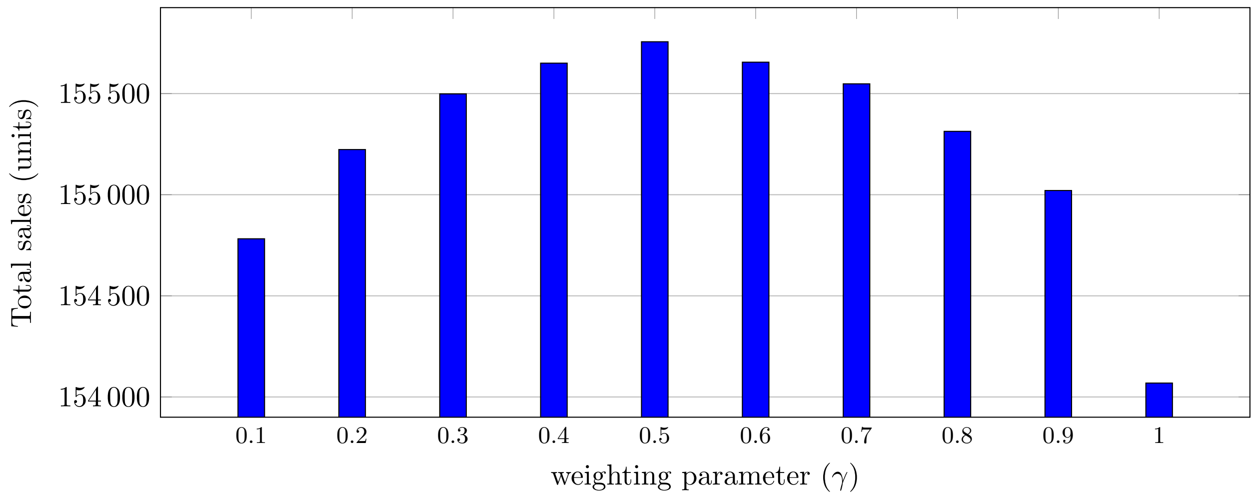

FIGURE 5.19: *Subclass W1 total sales comparison for varying weighting parameter (γ) values, where $\gamma = 1$ reflects static size profile sales.*

The change in total sales for each value of γ against static sales ($\gamma = 1$), is presented in Figure 5.20. The greatest percentage improvement occurs where dynamic size profile adjustments use $\gamma = 0.5$. Using this weighting parameter value means each store's size profile adjust to represent 50% of the historical size profile and 50% of the current size profile, at the time of allocation. Using $\gamma = 0.5$, total sales increase by 1.1%, which equates to an additional 1 687.4 units of stock that are sold, on average for Subclass W1, compared to static size profile sales.

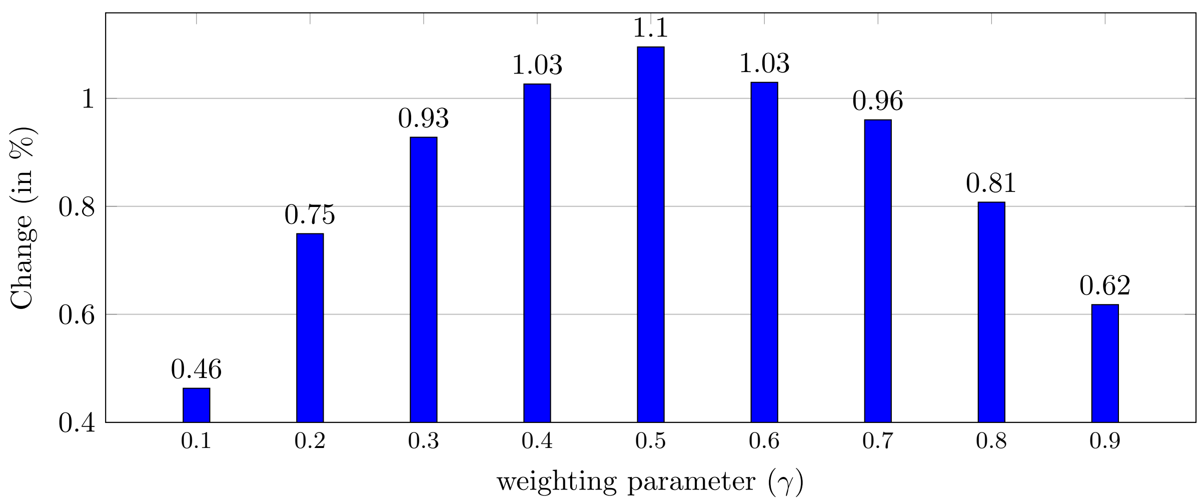

FIGURE 5.20: *Average percentage change in total sales for varying weighting parameter, γ values, compared to average static size profile ($\gamma = 1$) sales in Subclass W1.*

The second highest improvement in total sales occurs when $\gamma = 0.4$ or $\gamma = 0.6$ are used, increasing total sales by 1.03% on average for W1, compared to static size profile sales. The smallest percentage improvement in total sales is recorded using $\gamma = 0.1$, which results in a 0.46% increase (713.3 units, on average) in total sales. Subclass W1 records the largest actual sales quantity compared to the other subclasses considered in this study. A possible reason $\gamma = 0.1$ records an increase in total sales for W1 is due to the size (with regards to total inflow, thus sales) of the subclass. As it has been noted previously in this study that volatility of sales—characterised by smaller subclasses—favours larger γ values. Therefore, larger subclasses with more stability of sales, would favour smaller γ values.

5.3.2 System comparison

As identified above, simulating sales using $\gamma = 0.5$, results in the largest percentage improvement in total sales. Statistical analysis tests the difference between static size profile sales and dynamic size profile sales. Static size profile, using $\gamma = 1$ simulation output is classified as system one and dynamic size profile adjustment for $\gamma = 0.5$, is classified as system two. To determine the confidence interval for the difference in sales, each replication output from system two (dynamic system) is subtracted from the corresponding replication output from system one (static system).

The calculation of confidence intervals assumes the difference in means follows a normal distribution. Due to the small sample size ($n = 10$), two formal tests of normality, namely Shapiro-Wilk and Anderson-Darling test the null hypothesis against the alternative hypothesis. The null hypothesis states that the difference in means follow a normal distribution and the alternative hypothesis states that the data are not normally distributed. Results from these two formal tests of normality are obtained via Python 3.6.3 [32] and presented in Table 5.8. At a significance level of 0.05, the p-value obtained for both test are greater than the level of significance. The null hypothesis is not rejected and assuming normality, the calculation of confidence intervals may commence.

Test	Statistic	p value
Shapiro-Wilk	0.928	0.430
Anderson-Darling	0.317	0.684

TABLE 5.8: *Statistical test of normality for system comparisons at Subclass W1.*

A mean difference between the data series is $-1\,687.4$ units and the standard deviation is 146.3 units. The t-distribution has 9 degrees of freedom and an estimated 95% confidence interval for the difference in means is given by $[-1\,792.03, -1\,582.77]$, which is negative because of the calculation (static sales $-$ dynamic sales). The confidence interval lies below 0 and concludes the system with dynamic size profile adjustment using $\gamma = 0.5$ are performing better that the system with static size profiles.

Figure 5.20 presents the percentage change in total sales for each varying value of the weighting parameter. According to this bar plot, each value of γ results in a positive improvement in total sales, compared to the system of static size profiles. Testing the range of statistically significant γ values that have an effect on total sales, provides more information regarding the range of acceptable values that may be used when dynamically adjusting size profiles for Subclass W1.

The output generated when dynamically adjusting size profiles using $\gamma = 0.1$ is tested against the static system. The difference in data are normally distributed with a mean of -713.3 units and a standard deviation of 126.74 units. The estimated 95% confidence interval for the difference in means is given by $[-800.78, -625.82]$, confirming the dynamic system using $\gamma = 0.1$, performs better than the static system in generating sales. The difference in means for sales generated using $\gamma = 0.9$ is not normally distributed and the confidence interval cannot be computed. Intuitively, it makes sense that a value $\gamma = 0.9$ should not be used for Subclass W1, as the adjusted size profile is too small to make a significant difference in total sales. However, at a 0.05 level of significance, the difference in mean sales between the dynamic system using $\gamma = 0.8$ and the static system where $\gamma = 1$, are normally distributed. The mean difference is $-1\,244.4$ units and the standard deviation is 161.08 units. The estimated 95% confidence interval is given by $[-1\,359.63, -1\,129.17]$, concluding that dynamic size profile adjustments using $\gamma = 0.8$ is statistically different and performs better than the system of static size profiles. The range of statistically significant weighting parameter values that increase total sales for Subclass W1 is given by $\gamma = 0.1$–0.8.

Weekly analysis

Throughout the season eleven styles of Subclass W1 arrive at the DC from factories. Nine of the styles contain allocation data and they are sent in nine separate weeks. For the two styles where no allocation data is available, the Retailer's actual inflow quantities are sent and it is assumed that any adjustment would arrive at the same result. These two styles are sent to stores in weeks 3 and 4. The first style with allocation data arrives at the DC in week 5. As stores have received stock inflows previously in the season and sales have been simulated, size profiles are dynamically adjusted by considering current recorded sales performance. Thereafter, size profiles are dynamically adjusted in weeks 8, 9, 11, 13, 17, 18, 20 and 22.

This study is concerned with analysing the effect dynamic size profile adjustments have on total sales. Figure 5.21 plots the weekly sales on a company level for the two systems (static, where $\gamma = 1$ and dynamic, where $\gamma = 0.5$) under analysis in this section. The average simulated sales generated using static size profiles are plotted in blue and average sales generated via dynamic size profile adjustment using $\gamma = 0.5$, are plotted in red. This weekly comparison between the two systems leads to an understanding of the overall impact dynamic size profile adjustments have on weekly sales. The general pattern of the season's sales show that the majority of sales occur between week 9 and week 18, with a dip in week 15 and 16. Over this time span, the corresponding weekly bars indicate that total sales are higher when size profiles are dynamically adjusted. Meaning dynamic size profile adjustments respond to changing customer demand and improve the partitioning of fixed company size-mix into smaller size-mixes for stores.

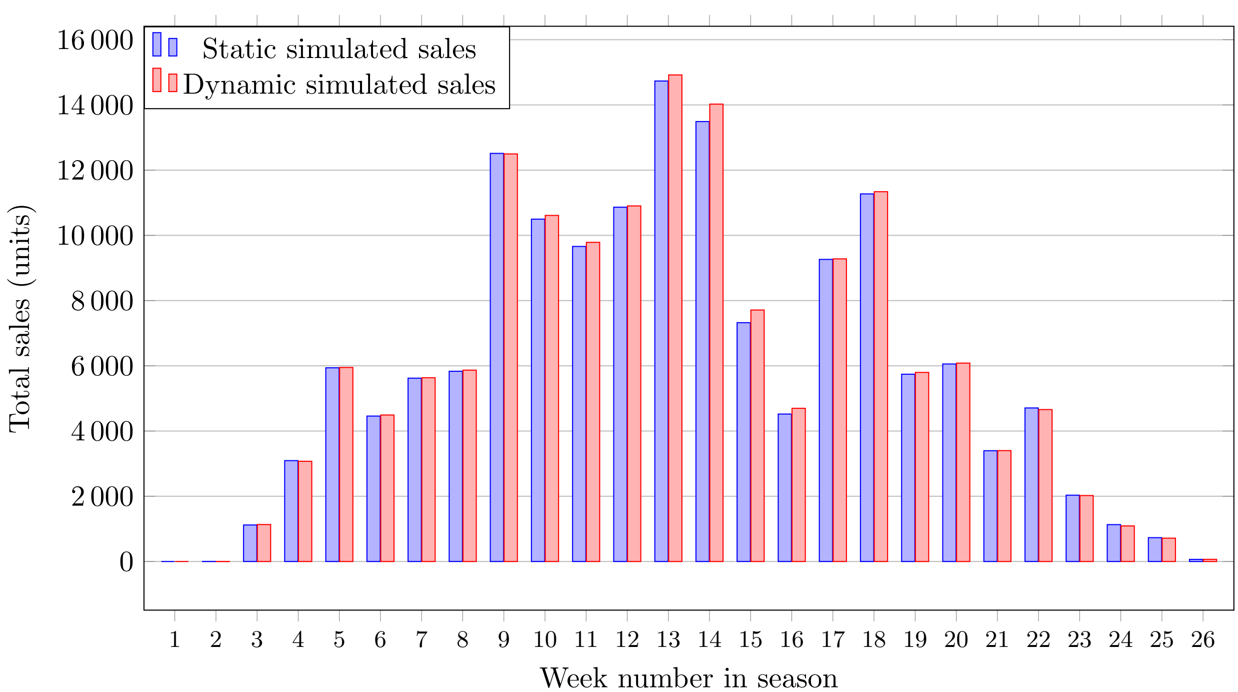

FIGURE 5.21: *Weekly comparison of opposing system simulated sales at Subclass W1.*

Figure 5.22 plots the weekly unit change in sales. Each week the average sales from the static system are subtracted from the dynamic system, visually presenting the weekly change experienced on a company level when size profiles are dynamically adjusted using $\gamma = 0.5$. Figure 5.21 and Figure 5.22 present the same system results in complimentary ways to visually facilitate the interpretation of the effect on total sales that dynamic size profile adjustments have on Subclass W1.

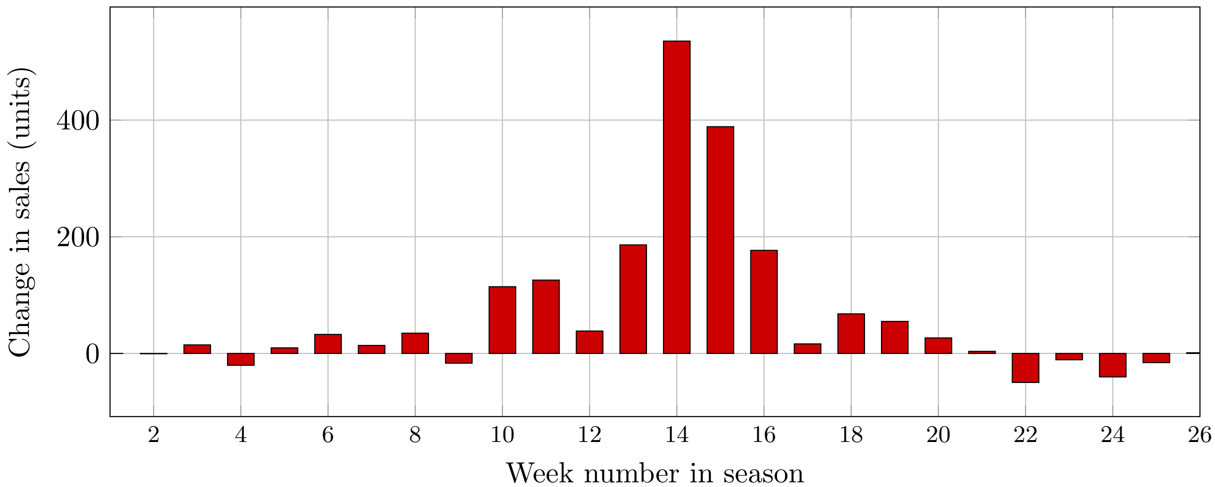

FIGURE 5.22: *Weekly difference in average sales generated using $\gamma = 0.5$ dynamically adjusting size profiles, compared to static size profile sales.*

Stock arrives at stores in week 3, allowing size profile adjustments to commence for the following weeks when styles arrive at the DC from factories, contain allocation data and only stores that meet the adjustment conditions receive a new adjusted size profile. The first style to arrive at the DC from factories with allocation data is in week 5, where sales increase on average by 9.6 units, compared to static size profile sales. Sales increase in 18 of the 24 weeks (excluding weeks 1 and 2, as no stock has arrived). The largest increase in total sales is recorded in week 14, where an additional 535.4 units are sold, on average. Figure 5.22 provides a clear indication of the change in sales and it is apparent that decreases in total sales are of a smaller quantity than the recorded weeks where sales increase as a result of dynamic size profile adjustments.

The evaluation framework presented in Figure 5.23 plots the weekly performance of static size profile system ($\gamma = 1$) against the dynamic size profile system, where $\gamma = 0.5$. Demand satisfaction is plotted along the x-axis (DCR) and shipment success is plotted along the y-axis (SSR). Static size profile performance is plotted by the blue squares and the performance of dynamically adjusting size profiles is plotted by the red diamonds. Each point on the figure represents the cumulative weekly performance to date, enabling a complete comparison between systems. As the calculations are cumulative, points move from left to right and performance measures close to 1 are ideal. Red dots visible to the right of corresponding blue dots indicate greater demand coverage, meaning sales are recorded where there is demand as a result of improved allocation and resulting in less shortages. Red dots above corresponding blue dots are an indication of increased sales due to an improvement of stock allocation amongst stores, meaning the amount of sales relative to inflows have improved.

The weekly performance for both systems appear to be relatively similar until week 10, where both points lie just below 0.6 on the y-axis. In week 10 the dynamic system (red diamond) shifts slightly to the right of the static system indicating an improvement in demand satisfaction. Meaning more demand is able to be fulfilled by available stock and therefore more sales occur.

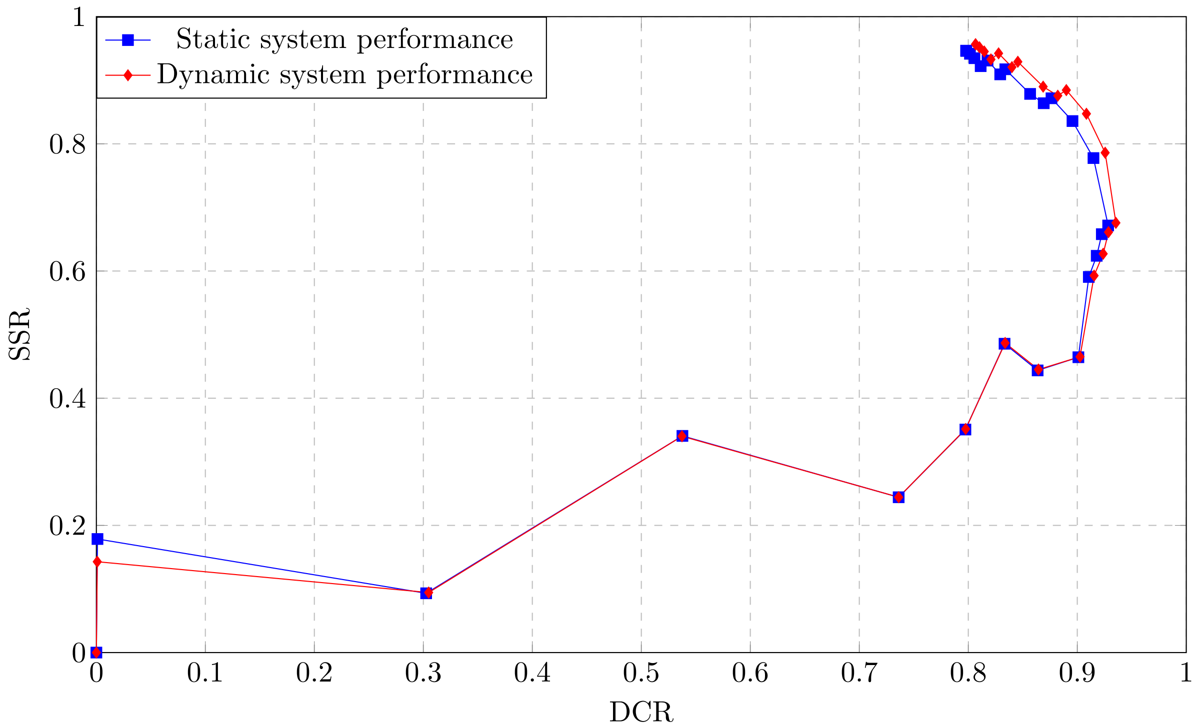

FIGURE 5.23: *Subclass W1 company evaluation framework of static system performance and dynamic system performance, where* $\gamma = 0.5$.

From week 10 onwards, the dynamic system remains closer to 1 on the x-axis than the static system (to the right of the blue squares) and steadily moves closer to 1 on the y-axis. Towards the end of the season, demand satisfaction decreases slightly compared to the previous weeks possibly due to overall less stock due to seasons end. However, red diamonds still remain to the right and above corresponding blue squares. Concluding less surplus stock at the end of the season and generally fewer weekly stockout situations, linked to the increased demand satisfaction.

Weekly analysis for stores

It is presumed that larger stores shall benefit more from size profile adjustments than smaller stores. The size of a store (in this study) is quantified by the total amount of inflow carried throughout the season. Total inflow for all 1 273 stores in this subclass, amounts to 162 816 units for the season. The minimum total inflow received at a store is 11 units, and the maximum total inflow at a store is 1 172 units. Analysis of inflows indicates that about 75% of stores receive 170 units or less for the season. These stores inflows constitute merely 37.27% of the total inflow for the subclass. The remaining stores inflow amounts to a total of 62.78% for the season, indicating the majority of stores in Subclass W1 are small.

Stores are categorised by relative inflow quartiles in Table 5.9. Category 1 includes stores where inflows are less than 35 units, Category 2 includes stores with inflow between 36 and 75 units and Category 3 contains stores with inflow that is between 76 to 170 units for the season. Category 4 contains large stores, where total inflow for the season is between 171 and 1 172 units.

Total sales for stores within each of these categories are affected by dynamic size profile adjustment. In comparison to total sales generated by the static system ($\gamma = 1$) when $\gamma = 0.5$ is

Category	% of stores	Range of inflow (MIN-MAX)	% of total inflow
1	23.96%	11–35	3.76%
2	25.37%	36–75	10.24%
3	25.37%	76–170	23.23%
4	25.29%	171–1 172	62.78%

TABLE 5.9: *Subclass W1 categorisation of stores given inflows.*

used, sales increase on average by 0.81% or 42.9 units for stores in Category 1 and decrease on average by 1.11% and 1.38% or 168.9 and 496.7 units for stores in Category 2 and 3, respectively. Total sales for stores in Category 4 increase on average by 2.37% or 2 310.1 units. An evaluation framework of sales performance using $\gamma = 0.5$ for stores in each category of Subclass W1 is available in Appendix A.3.1.

Analysis of the effect extreme weighting parameter values, $\gamma = 0.1$ and $\gamma = 0.9$ have on stores with varying sizes are presented in this section. Using a parameter of $\gamma = 0.1$, total sales in Category 1, 2 and 3 decrease on average by 1.49%, 3.58% and 2.06% (or 87.9, 542.4 and 743.2 units), respectively. Total sales for stores in Category 4 increase on average by 2.13% or 2 077.8 units for the season when dynamically adjusting size profiles using $\gamma = 0.1$, compared to static size profile sales.

The smallest stores (in terms of total inflow) are in Category 1 and the largest stores are in Category 4. To analyse the effect of $\gamma = 0.1$, evaluation frameworks of stores in Category 1 and 4 are presented in Figure 5.24 and Figure 5.25, respectively. Red diamonds indicate the weekly effect of using $\gamma = 0.1$ and blue squares indicate static size profile performance. The evaluation frameworks stores in Category 2 and 3 are available in Appendix A.3.2, as well as the evaluation framework on a company level.

The performance of stores in Category 1 (Figure 5.24) record a decrease in demand satisfaction, compared to static size profile performance (red diamonds to the left for blue squares). Stores in Category 4 (Figure 5.25) indicate an increase in demand satisfaction from week 8 onwards Meaning, large stores respond well to size profile adjustments using $\gamma = 0.1$ and small stores are negatively affected. However, the large adjustment of size profiles from using $\gamma = 0.1$ result in 232.3 less units sold, on average, compared to total sales recorded using $\gamma = 0.5$.

On the other hand, dynamically adjusting size profiles using $\gamma = 0.9$ means 90% of the adjusted size profile comprises of historical size profile (generated by the Retailer) and 10% comprises of the current size profile (as recorded to date). Total sales for stores in Category 1, 2 and 3 decrease on average by 0.08%, 2.00% and 2.18% (or 4.4, 302.5 and 786 units), respectively, compared to static size profile sales. Stores in Category 4 increase by 2.10% or 2 044.9 units, compared to total sales generated when size profiles remain static using the $\gamma = 1$.

Small stores record a smaller decrease, and large stores record a smaller increase in total sales from the use of $\gamma = 0.9$, compared to an adjustment in the opposite direction ($\gamma = 0.1$). For comparison, Figure 5.26 and Figure 5.27 present the evaluation frameworks for stores in Category 1 and 4, respectively. The evaluation framework for stores in Category 2 and Category 3 are available in Appendix A.3.3, as well as the evaluation framework on a company level.

As expected, stores in Category 1 indicate weekly red diamonds that are practically on top of the blue squares throughout the season. The small movement along both axes is expected due to the small adjustment declared by $\gamma = 0.9$. For stores in Category 1, the use of a large weighting parameter such as $\gamma = 0.9$ records better performance than when a small weighting parameter

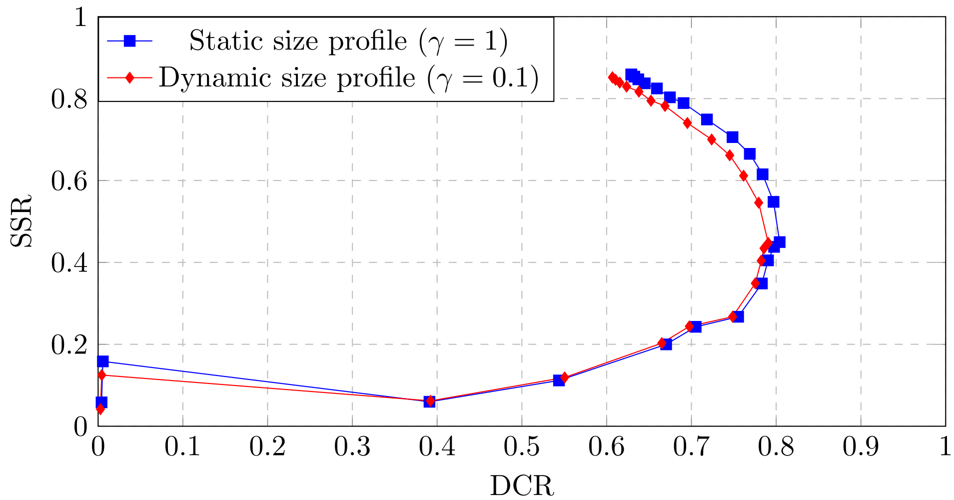

FIGURE 5.24: *Subclass W1 evaluation framework of static system performance and dynamic system performance, where* $\gamma = 0.1$*. Considering stores in Category 1.*

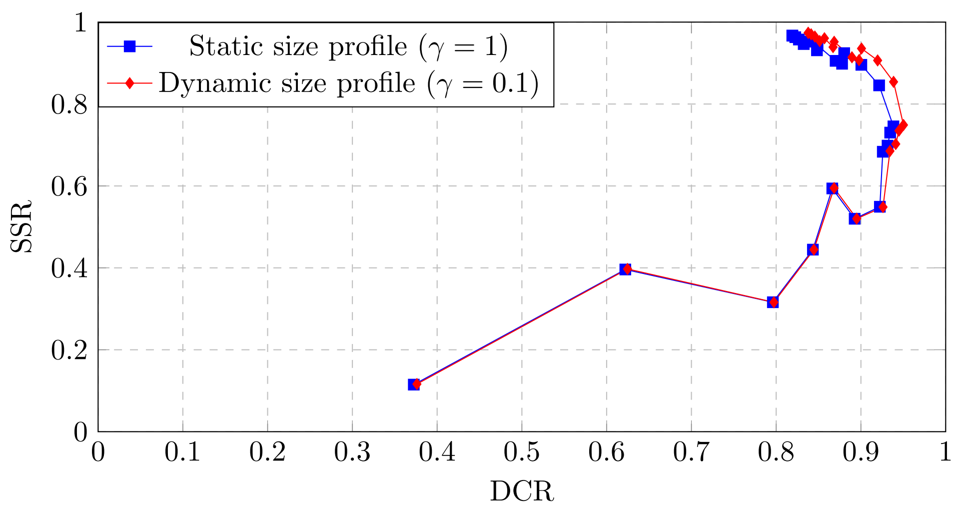

FIGURE 5.25: *Subclass W1 evaluation framework of static system performance and dynamic system performance, where* $\gamma = 0.1$*. Considering stores in Category 4.*

such as $\gamma = 0.1$ (presented in Figure 5.24) is used. However, the use of $\gamma = 0.5$ (presented in Figure A.21) outperforms the use of $\gamma = 0.9$ for stores in Category 1 (small stores) in Subclass W1. Therefore, even though the total inflow contribution for stores in this category is merely 3.76%, the performance of sales is better where $\gamma = 0.5$ or a value close to it is used.

Considering stores in Category 4, Figure 5.27 presents the weekly performance of dynamic size profile adjustments where $\gamma = 0.9$ is used. Again red diamonds are close to blue squares, with a slight rightwards deviation once points are above 0.6 on the y-axis, until season's end. This movement indicates that the use of $\gamma = 0.9$ makes a slight improvement in total sales for stores in Category 4 with regards to demand satisfaction. Compared to dynamic size profile adjustments using $\gamma = 0.5$, on average the increase in total sales using $\gamma = 0.9$ are 265.2 units less than when $\gamma = 0.5$ is used. In conclusion, the use of a large value of the weighting parameter (*i.e.* $\gamma = 0.9$) is not recommended for Subclass W1 as there is better performance in total sales when a smaller weighting parameter value (*i.e.* $\gamma = 0.5$) is used.

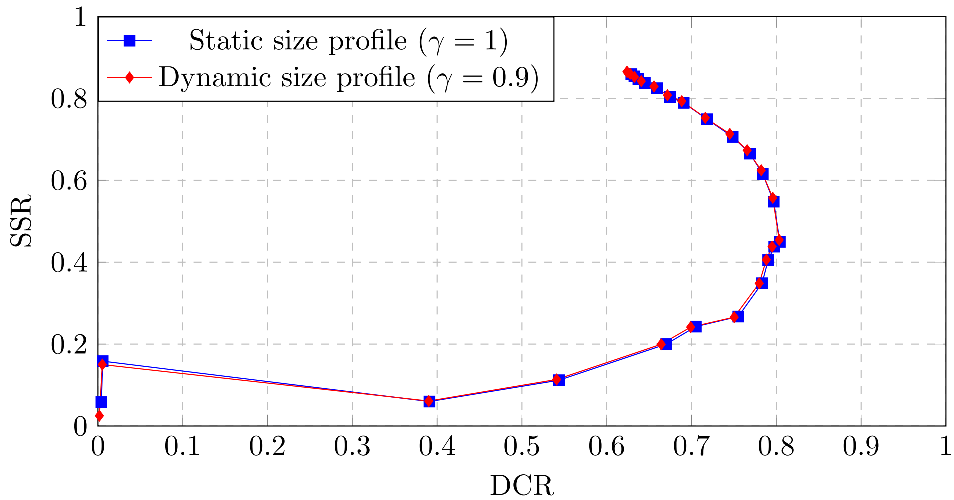

FIGURE 5.26: *Subclass W1 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.9$. Considering stores in Category 1.*

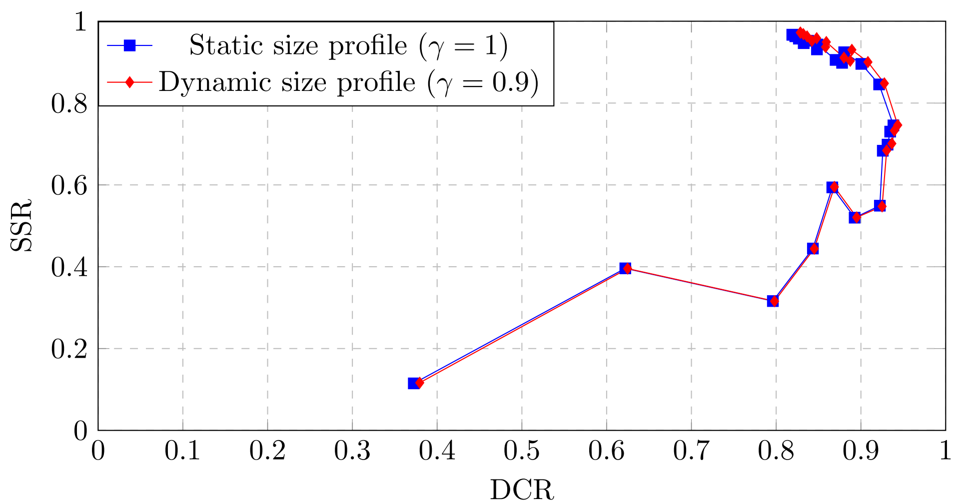

FIGURE 5.27: *Subclass W1 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.9$. Considering stores in Category 4.*

5.4 Subclass W2

Subclass W2 is the second winter subclass considered in this study, consisting on exactly 26 weeks within the season 08 February 2014 – 02 August 2014. There are 950 stores in this subclass and each store receives six various sizes of the product (Ladies spun poly jackets), from European size 32 to size 42. Subclass W2 comprises of a total of 12 styles for the season, one style has no allocation data and the actual inflows, received by the Retailer is used in place of the solutions that would have been generated by the size-mix allocation. If weeks range from 1–26, the first allocation decisions are finalised in week 3. Two styles with allocation data are set to arrive on this date, static size profiles are used to finalise allocation decisions as no sales have been recorded to date for this subclass. The remaining 9 styles with allocation data are allocated over four various weeks throughout the season, where size profiles are dynamically adjusted for stores that meet the adjustment criteria (sales have been recorded to date).

The effect of dynamically adjusting size profiles are analysed in this section by simulating weekly sales using the simulation model described in Chapter 3. Simulation input parameters validated and verified to provide sufficient accuracy are presented in §4.3.4. These parameters are used in the simulation of sales when dynamic size profile adjustments are implemented. The simulation model with static size profiles ($\gamma = 1$) generates output which is validated in §4.4.4 to be sufficiently close to the real system sales.

This section analyses the effect of dynamically adjusting size profiles. Sensitivity analysis on the effects of varying the weighting parameter value is available in §5.4.1. Statistical analysis follows in §5.4.2 to determine whether observed differences in simulated sales are statistically significant. Thereafter, weekly analysis on a company level and then on a store level are analysed, providing insight into the weekly effect dynamic size profile adjustments have on total sales.

5.4.1 Sensitivity analysis

Sensitivity analysis is performed to analyse the effect each value of weighting parameter, γ, has on total sales. The simulation model is replicated 10 times for each value of γ and the average total sales are reported on to ensure reliability in results. Table 5.10 provides the total simulated sales generated when dynamically adjusting size profiles using various γ values. The final row presents the total sales from the validated simulation model, where size profiles remain static by using $\gamma = 1$.

Weighting parameter	Simulated sales (avg)
$\gamma = 0.1$	64 628
$\gamma = 0.2$	64 885.7
$\gamma = 0.3$	65 626.1
$\gamma = 0.4$	66 299
$\gamma = 0.5$	66 652.3
$\gamma = 0.6$	67 064.5
$\gamma = 0.7$	67 135.8
$\gamma = 0.8$	67 114.9
$\gamma = 0.9$	66 761.9
$\gamma = 1$	66 065.6

Table 5.10: *Sensitivity analysis of W2 simulated total average sales for varying γ values, including static size profile sales where $\gamma = 1$.*

Figure 5.28 presents the total sales generated by the simulation model for varying values of γ. The final bar represents total sales generated by the simulation model where size profiles remain static throughout the season by using $\gamma = 1$.

The change in total sales is available in Figure 5.29 for each value of γ, compared to sales generated when size profiles remain static ($\gamma = 1$). Weighting parameters within the range $\gamma = 0.4$–0.9 report a positive percentage change in total sales, compared to static size profile sales. The largest percentage improvement is recorded where $\gamma = 0.7$ is used. Using $\gamma = 0.7$, total sales increase by 1.62%, compared to sales recorded by the static system. This percentage improvement results in an average increase in total sales of 1 070.2 units for the season. The use of $\gamma = 0.8$ reports the second highest percentage improvement in total sales, followed by $\gamma = 0.7$, $\gamma = 0.9$, $\gamma = 0.5$ and lastly $\gamma = 0.4$. The use of $\gamma = 0.1$–0.3 record a decrease in total sales, compared to sales generated when size profiles remain static.

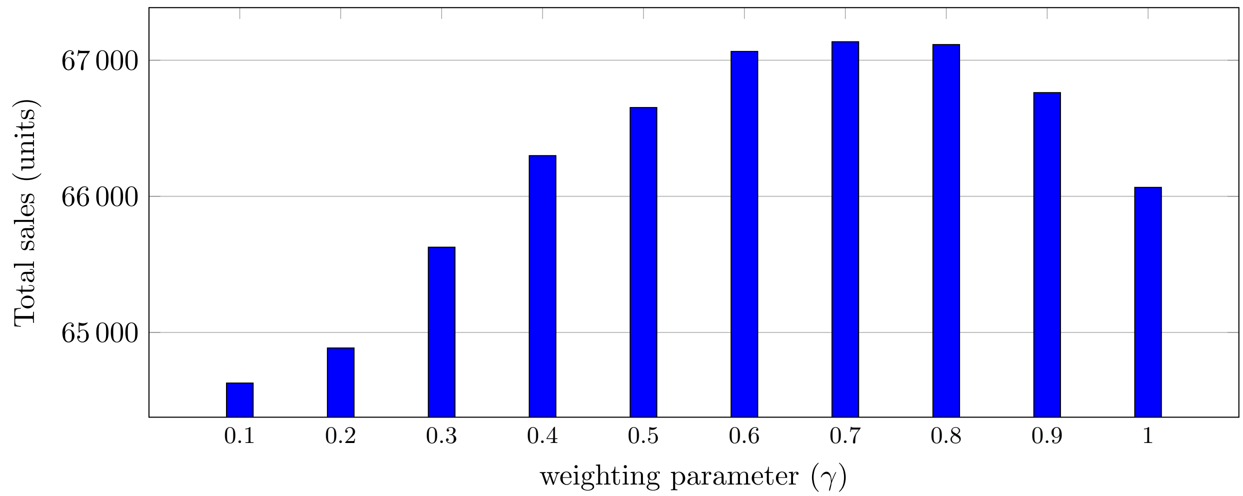

FIGURE 5.28: *Subclass W2 total sales comparison for varying weighting parameter (γ) values, where $\gamma = 1$ reflects static size profile sales.*

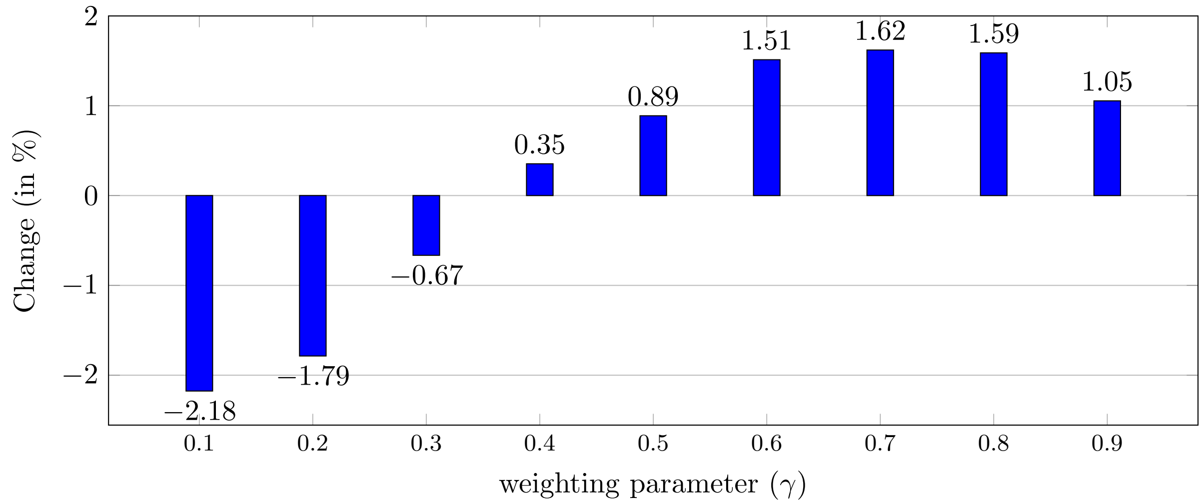

FIGURE 5.29: *Subclass W2 average percentage change in total sales for varying weighting parameter, γ values, compared to actual total sales.*

5.4.2 System comparison

The use of $\gamma = 0.7$ in dynamic size profile adjustments results in an average total sales increase of 1.62%, which is the largest recorded improvement on total sales for Subclass W2. Confidence intervals of the difference in means, a classical method of system comparison; is performed on the difference between static size profile simulated sales ($\gamma = 1$) and dynamically adjusted size profile sales, where $\gamma = 0.7$.

Normality of the differences in means must first be determined before confidence intervals may be calculated. Two formal tests of normality are conducted on this small sample size ($n = 10$). Results are obtained via Python 3.6.3 [32] and presented in Table 5.11.

At a significance level of $\alpha = 0.05$, both tests do not reject the null hypothesis of normality and it is assumed that the difference in total sales are normally distributed. The mean difference in total sales is $-1\,070.2$ units with a standard deviation of 425.56 units. The t-distribution has 9 degrees of freedom and at a significance level of 0.05, an estimated 95% confidence interval for

Test	Statistic	p value
Shapiro-Wilk	0.961	0.793
Anderson-Darling	0.293	0.684

Table 5.11: *Statistical test for normality of system comparisons on Subclass W2.*

the difference in systems is given by $[-1\,374.41, -765.99]$, where the negative sign is due to the calculation of differences (static − dynamic). The confidence interval lies below zero, concluding the difference in means is statistically different and the dynamic adjustment of size profiles using $\gamma = 0.7$ performs better than the system with static size profiles (indicated by the negative sign in the confidence interval).

The use of weighting parameter values $\gamma = 0.4$–0.9 records percentage increases in total sales, compared to static size profile sales, presented in Figure 5.29. This range of weighting parameter values are tested for statistical significance against the static system for Subclass W2. At a 0.05 level of significance, the difference in means for this range of weighting parameter values fail to reject the null hypothesis and it is assumed the data are normally distributed. The mean difference between sales generated using $\gamma = 0.4$ and $\gamma = 0.9$ against static size profiles sales ($\gamma = 1$) is -233.4 and -696.3 units, respectively. The standard deviation in difference is 291.16 units for $\gamma = 0.4$, and 354.34 units for $\gamma = 0.9$. The estimated 95% confidence interval for the difference in means of $\gamma = 0.4$ and the static system, is given by $[-441.68, -25.12]$, concluding the dynamic system for this value of the weighting parameter performs better than the static system. Considering the use of $\gamma = 0.9$, an estimated 95% confidence interval of dynamic system sales subtracted from static system sales ($\gamma = 1$), is given by $[-949.78, -442.82]$, concluding the dynamic system where $\gamma = 0.9$ performs better than the static system. These confidence intervals indicate that the range of weighting parameter values, $\gamma = 0.4$–0.9 are statistically different from the static system. It is concluded that the use of a weighting parameter within this range results in a positive change in total sales, where the use of $\gamma = 0.7$ results in the largest percentage improvement.

Weekly analysis

Subclass W2 receives a total of 12 styles throughout the season, one of which has no allocation data and the Retailer's calculated inflows are used under the assumption that any adjustment would generate sufficiently similar results. Therefore, 11 styles containing allocation data arrive at the DC from factories in weeks 3, 7, 11, 15 and 19 in the season.

Figure 5.30 presents the weekly total sales generated by the static system ($\gamma = 1$), indicated in blue. Weekly sales generated from the dynamic adjustment of size profile using $\gamma = 0.7$, are presented in red. Stock arrives at the DC in week 3 for two styles with allocation data available, as this is the first allocation no adjustments to the size profiles are made. Stock is allocated using the static size profile, determined by the Retailer using historical sales data, months before the selling season starts. The following style for W2 arrives in week 7 and as sales have been recorded in previous weeks, size profiles are dynamically adjusted.

The pattern of weekly sales for Subclass W2 are below 1 300 units on average until week 11 when they start increasing, reaching a maximum in week 18 followed by a decline until the end of the season. Figure 5.31 presents the change in weekly sales as experienced by the dynamic adjustment of size profiles using $\gamma = 0.7$. The pattern of unit change is similar to the total weekly sales, with week 18 recording the largest using improvement, averaging 207.4 units. Week 20

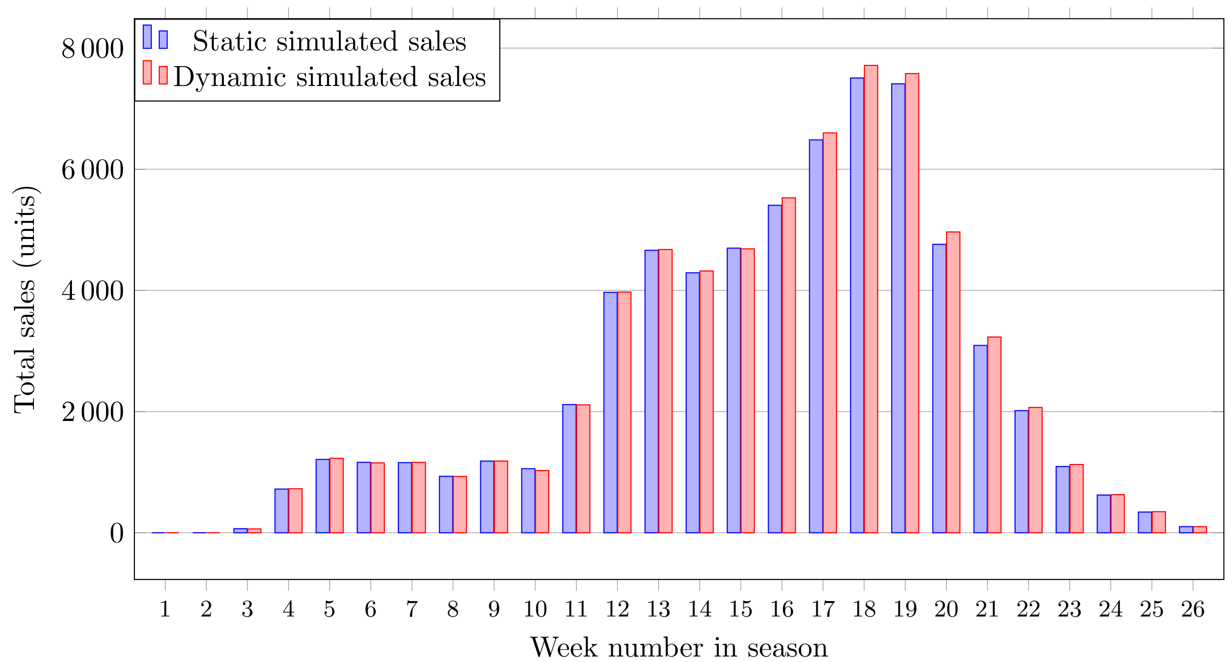

FIGURE 5.30: *Weekly comparison of opposing system simulated sales at Subclass W2.*

records the second largest improvement in total sales with an additional 204.3 units sold on average, from the use of dynamic size profile adjustments, where $\gamma = 0.7$. The overall plot indicates a greater frequency and magnitude of increasing weekly sales compared to decreasing sales, generated from the dynamic adjustment of size profiles.

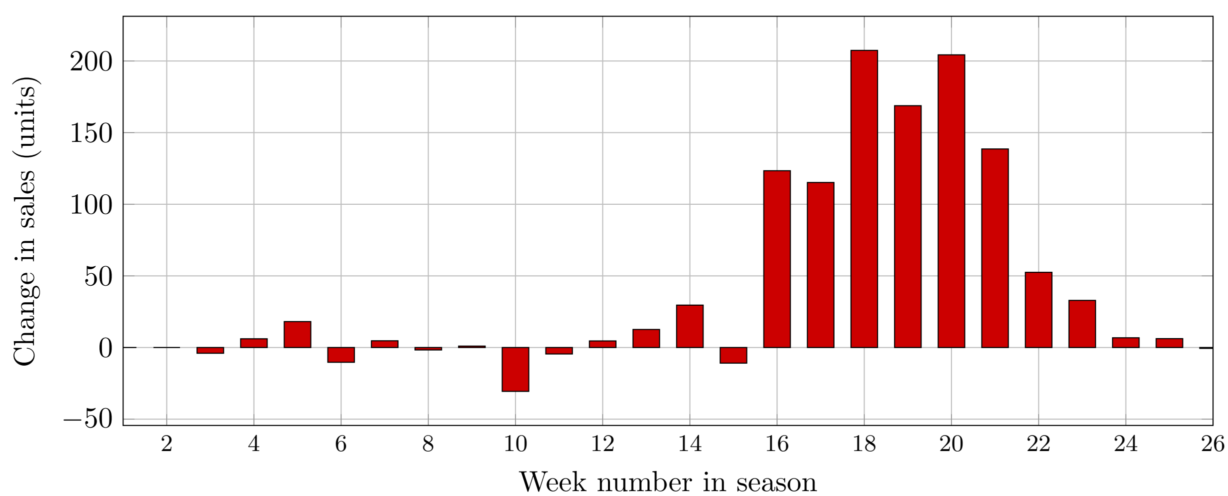

FIGURE 5.31: *Weekly difference in average sales generated using $\gamma = 0.7$ dynamically adjusting size profiles, compared to static size profile sales.*

An evaluation framework of the company (all stores and sizes) performance is presented in Figure 5.32, where each point represents a week in the season. The blue squares reflect the performance when size profiles remain static and the red diamonds reflect the performance when size profiles dynamically adjust using $\gamma = 0.7$. Red diamonds to the right and above corresponding blue squares imply an improvement in the system of dynamic size profile adjustments. Using the evaluation framework to analyse the effect of dynamic size profile adjustments against the static system not only considers the total sales, but also the inflow and demand as recorded by the simulation model of the two systems under consideration.

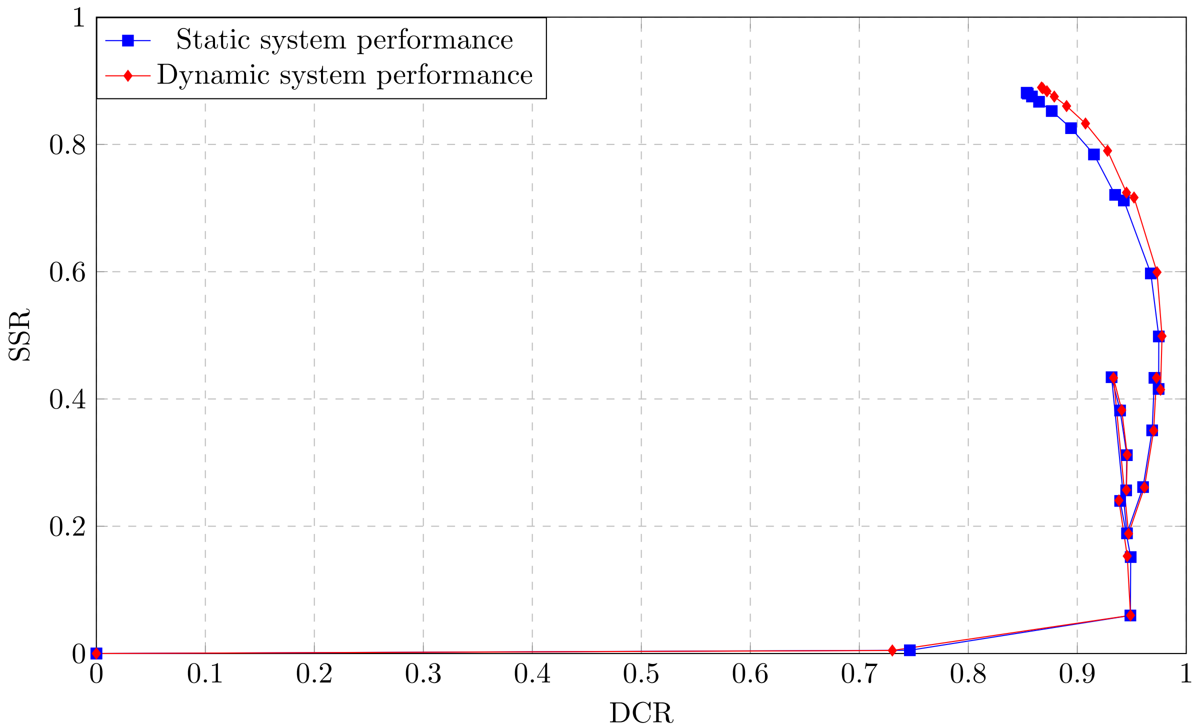

FIGURE 5.32: *Subclass W2 company evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.7$.*

The plot of both these lines are relatively similar until week 16 when red diamonds begin to diverge to the right of blue squares, indicating an increasing proportion of demand that is able to convert into sales due to improved allocation availability. The point of diversion corresponds with the plot presenting unit change in total weekly sales. Until week 16, the change in sales recorded by the dynamic system are small and oscillate between increasing and decreasing amounts. However, from week 16 total weekly sales recorded by the dynamic system are much larger than the static size profile system.

The evaluation framework indicates that at the end of the season, total sales generated by the dynamic size profile adjustment system using $\gamma = 0.7$ results in less leftover stock. The amount of demand that was able to convert into sales has increased (closer to 1 along the x-axis) weekly due to allocation that is able to send stock where there is currently demand. As there is only a set amount of stock available for allocation throughout the season, only a slight upwards move along the y-axis is able to occur, however this slight increase results in less leftover stock at season's end.

Weekly store analysis

Analysis of total inflows for the 950 stores that receive stock of Subclass W2 specify that a minimum inflow of 10 units are received by a store for the season, and a maximum of 481 units are received by another store for the season. The total inflow for the season amounts to 75 618 units and the categorisation of stores according to inflow percentiles are available in Table 5.12.

Considering total sales generated by the dynamic system using $\gamma = 0.7$, compared to sales generated by the static system ($\gamma = 1$), stores in each category increase on average. Stores in

Category	% of stores	Range of inflow (MIN-MAX)	% of total inflow
1	23.58%	10–32	5.52%
2	26%	33–59	14.22%
3	25.37%	60–112	26.05%
4	25.05%	113–481	54.21%

TABLE 5.12: *Subclass W2 categorisation of stores given inflows.*

Category 1, 2, 3 and 4 increase by 1.46%, 1.55%, 1.87% and 1.54%, respectively. The percentage increase equates to an average 46.3, 135.7, 316.5 and 571.7 unit increase for stores in Category 1, 2, 3 and 4, respectively. Each category's evaluation framework of the effect on sales using $\gamma = 0.7$ are available in Appendix A.4.1.

When dynamic size profile adjustments are performed using $\gamma = 0.1$, total sales decrease for stores in each category. On average total sales decrease by 5.49%, 6.40%, 3.15% and 0.46% for stores in Category 1, 2, 3 and 4, respectively. The percentage decrease equates to an average 173.7, 559.1, 534 and 170.8 units of Subclass W2 stock that are unsold at the end of the season. Stores in Category 1 and 4 reflect the smallest and largest stores in the subclass, respectively. An evaluation framework for stores in Category 1 is presented in Figure 5.33 and Figure 5.34 presents the evaluation framework for stores in Category 4. The evaluation frameworks for stores in Category 2 and 3, as well as an evaluation framework of the company are available in Appendix A.4.2. The evaluation frameworks contain a weekly comparison between the dynamic system, in red diamonds and static system ($\gamma = 1$) in blue squares. When red diamonds are to the right and above corresponding blue squares, it is implied the sales improvement is due to dynamically adjusting size profiles.

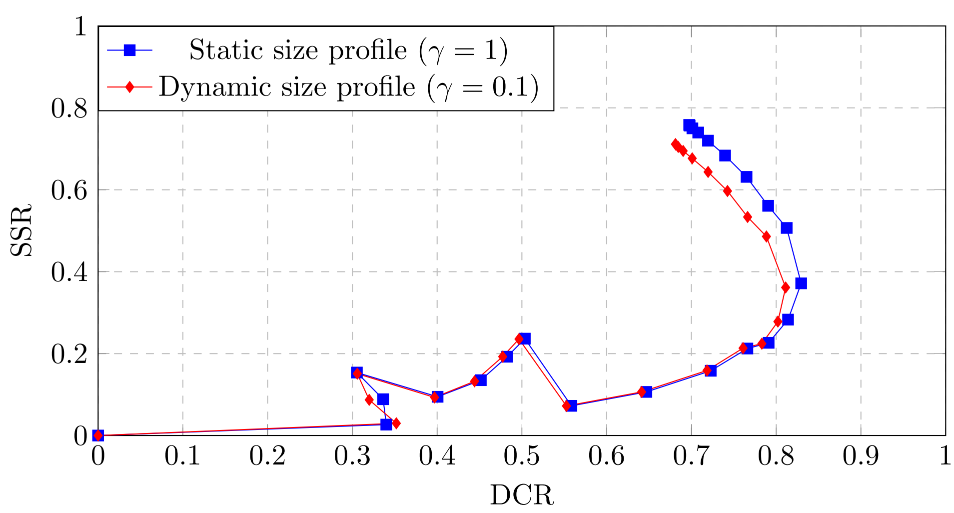

FIGURE 5.33: *Subclass W2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.1$. Considering stores in Category 1.*

In Figure 5.33 the dynamic system where $\gamma = 0.1$ is used, indicates sales for stores in Category 1 are worse than when size profiles remain static (red diamonds to the left and below corresponding blue squares). A significant diversion to the left is recorded from week 15 until season's end. Therefore, the use of $\gamma = 0.1$ for dynamic adjustment of size profiles is not recommended for small stores in Subclass W2. Figure 5.34, present Subclass W2's large stores (with regards to inflow) grouped into Category 4. A slight leftwards shift of red diamonds weekly are recorded

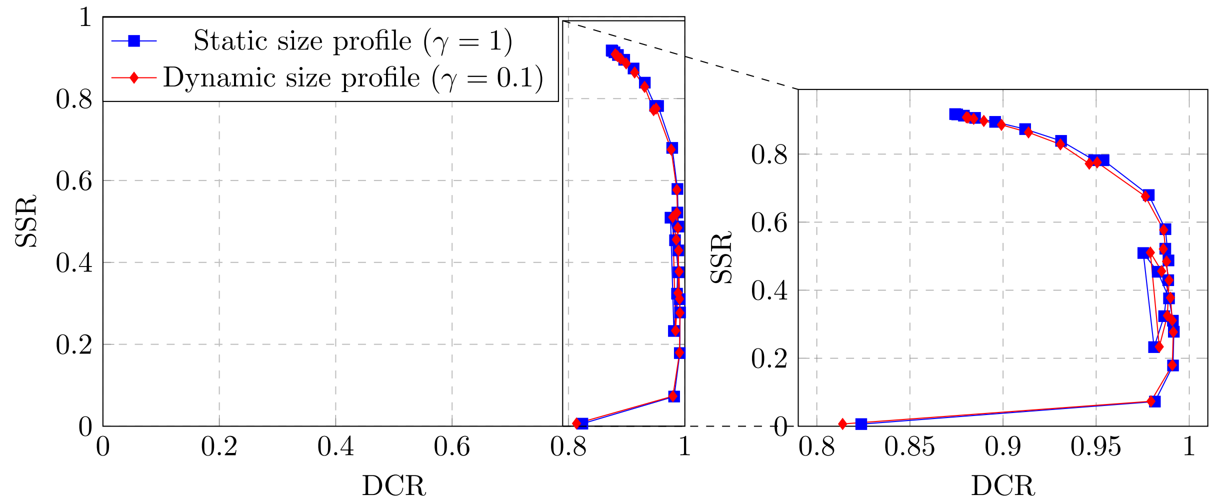

FIGURE 5.34: *Subclass W2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.1$. Considering stores in Category 4.*

and lower overall sales relative to inflow (along the y-axis) at season's end. Meaning, even large stores where total inflow is between 113–481 units do not respond well to dynamic adjustment using $\gamma = 0.1$.

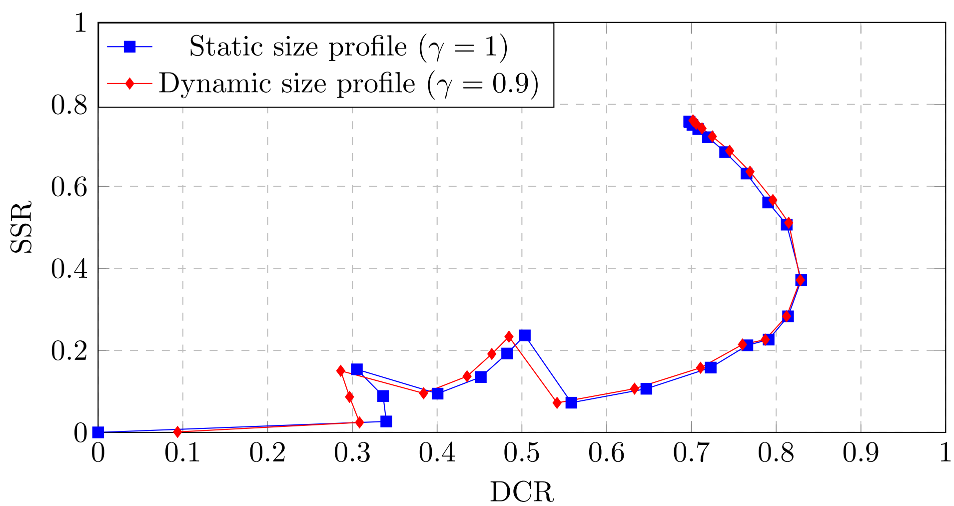

FIGURE 5.35: .
Subclass W2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.9$. Considering stores in Category 1.

The effect of dynamically adjusting size profiles using a large value of the weighting parameter, $\gamma = 0.9$ results in an average increase in total sales for stores within each category. Stores in Category 1, 2, 3 and 4 record an average increase of 0.33%, 1.53%, 1.14% and 0.96%, respectively, compared to total sales recorded by the static system ($\gamma = 1$). The percentage increase is equivalent to an additional 10.5, 133.9, 193.4 and 358.5 units that are sold from the use of dynamically adjusting size profiles using $\gamma = 0.9$, compared to static size profile sales. For comparability, stores in Category 1 and 4 are presented in Figure 5.35 and Figure 5.36, respectively. Evaluation frameworks for stores in Category 2 and 3 are available in Appendix A.4.3, as well as an evaluation framework of the effect on a company level.

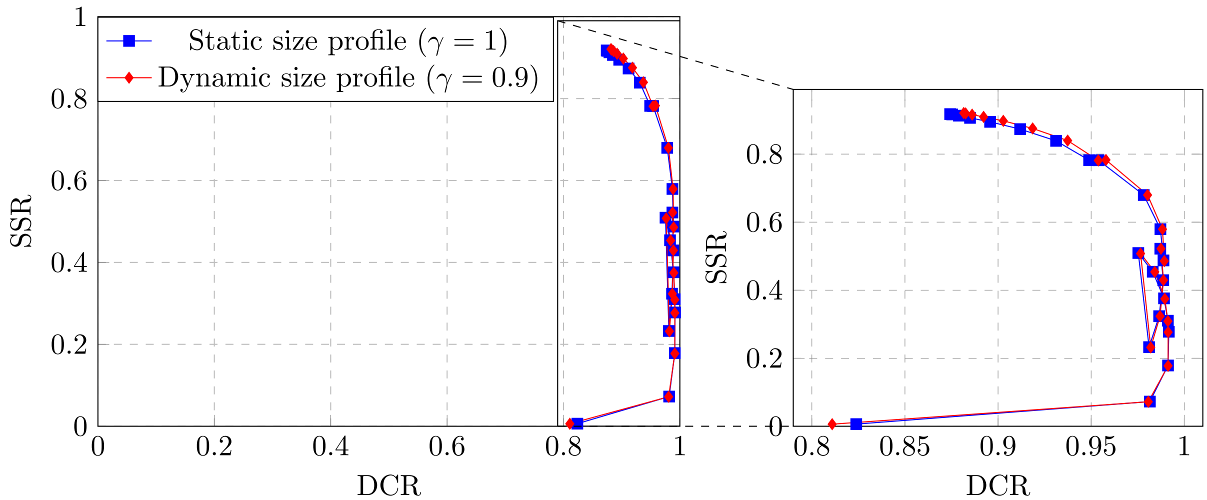

FIGURE 5.36: *Subclass W2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.9$. Considering stores in Category 4.*

Considering the evaluation framework for stores in Category 1 (Figure 5.35), demand satisfaction is marginally better than the static system towards the end of the season. At season's end, the red diamond is just peeking over the blue square and it is inferred that stores sales performance is better, resulting in less left over stock. However, the increase in total sales achieved from using $\gamma = 0.9$ are inferior to total sales generated when dynamic size profile adjustments are performed using the chosen value, $\gamma = 0.7$. Stores in Category 4 are presented in Figure 5.36, it is evident from the evaluation framework that dynamic size profile adjustments using $\gamma = 0.9$ are hardly better in terms of demand satisfaction than the static system sales and at season's end the improvement is barely noticeable. In conclusion, use of $\gamma = 0.1$ leads to an overall decrease in total sales and the increase in total sales generated from the use of $\gamma = 0.9$, is inferior compared to dynamic size profile adjustment using the chosen value, $\gamma = 0.7$.

5.5 Summary of results

The effect of dynamic size profile adjustments are analysed in this chapter. Size profile adjustments determine an updated size profile for stores at the time of allocation, reflecting a balance between historic and current sales at a store. This study utilises weekly recorded current sales to adjust size profiles, enabling better response during allocation to changing customer demand throughout the season. Two summer and two winter subclasses are considered and output generated from the sales simulation with dynamic size profile adjustments are reported on. The subclasses are simulated independently of one another.

Sensitivity analysis on experimentation of γ value variation is performed for each of the subclasses. Table 5.13 presents a summary of the results for each subclass considered in this study. Weighting parameter values that increase total sales by the largest percentage, compared to static size profile sales are selected as the "chosen γ", for each subclass. Statistical analysis is performed on the difference in sales generated by keeping size profiles static and sales generated by dynamically adjusting size profiles, using the "chosen γ". Formal tests of normality confirm the difference in sales are normally distributed and confidence intervals conclude the system of dynamic size profile adjustment perform better than the system where size profiles are kept static. A range of statistically significant γ values that generate better sales than the static system are identified for each subclass, presented in the final column.

Subclass	Chosen γ	Sales change (%)	Sales change (units)	Standard deviation	Range of γ
S1	$\gamma = 0.7$	0.94%	837.2	255.7	$\gamma = 0.3$–0.9
S2	$\gamma = 0.8$	2.17%	275.4	164.4	$\gamma = 0.4$–0.9
W1	$\gamma = 0.5$	1.1%	1 687.4	146.3	$\gamma = 0.1$–0.8
W2	$\gamma = 0.7$	1.62%	1 070.2	425.6	$\gamma = 0.4$–0.9

TABLE 5.13: *Summary of dynamic size profile adjustment results for each of the four subclasses.*

Weekly analysis on the effect of dynamic size profile adjustment using the "chosen γ", delineate an average increase in sales for 71–76% of the weeks where sales are recorded, amongst subclasses. Furthermore, the weekly change in sales follows the general pattern of weekly sales throughout the season. Generally sales are lower at the selling season's start and depending on the subclass, sales either increase around the middle or towards the end of the selling season. For example, summer subclasses (*i.e.* S1 and S2) record increased weekly sales towards the selling season's end, corresponding to Christmas sales/demand. Analysis of the change in sales achieved from the dynamic adjustment of size profiles indicate the weeks surrounding Christmas, record the highest increase in weekly sales. For each subclass considered in this study, weeks with historically higher sales record an even larger increase in total sales when size profiles are dynamically adjusted (for each "chosen γ"). Dynamic size profile adjustments reflect current sales performance of sizes within stores. Based on the pattern of weekly change in total sales, an improvement in the process of partitioning a fixed company size-mix into smaller size-mixes for stores is inferred.

It is deduced from the analysis of store categorisation that the use of extreme weighting parameter values (*i.e.* $\gamma = 0.1$ and $\gamma = 0.9$) are inadvisable. The Retailer's stores are mainly considered small (with regards to inflow). Sales are often more volatile in smaller stores and the use of a weighting parameter which is too small (*i.e.* $\gamma = 0.1$) has been shown to generate a negative change in total sales. Smaller γ values place an emphasis on current sales, which in the case of volatile sales, results in an overcompensation. Regardless of store size, the use of a weighting parameter which is too large (*i.e.* $\gamma = 0.9$) generates an inadequate increase in total sales, due to a relatively consistent size profile that is reflective of historical sales.

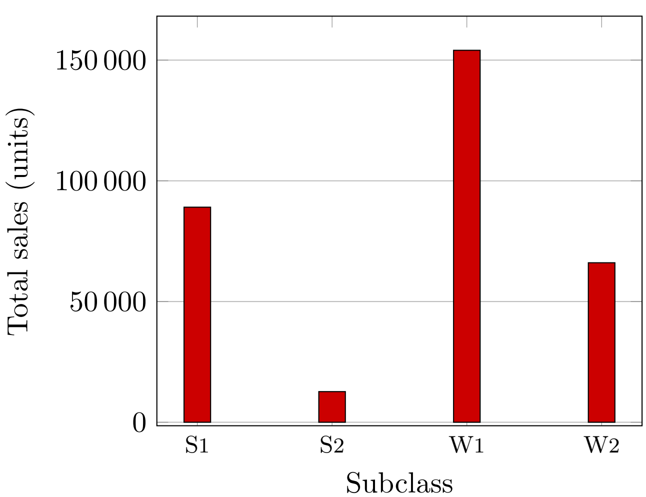

FIGURE 5.37: *Total static sales for subclasses.*

Figure 5.37 presents the total static sales for each subclass considered in this study. It is presumed subclasses with smaller total sales perform better when larger values of the weighting parameter are used, and vice versa. Subclass W1 records the highest total static sales for the season. Referring to Table 5.13, the smallest "chosen γ" is chosen for W1 ($\gamma = 0.5$) as well as the widest range of γ values ($\gamma = 0.1$–0.8). On the other hand, Subclass S2 records the lowest total static sales, the largest "chosen γ" ($\gamma = 0.8$) and the narrowest range of γ values ($\gamma = 0.4$–0.9).

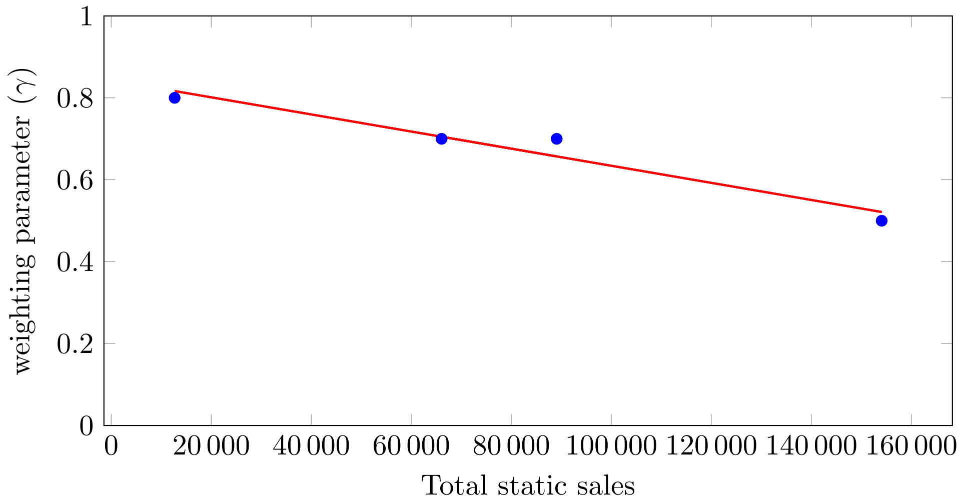

FIGURE 5.38: *Scatter plot of the correlation between total static sales and the weighting parameter value, as chosen amongst the four subclasses.*

Figure 5.38 presents a scatter plot of the correlation between total static sales and the corresponding chosen γ value across subclasses. The correlation coefficient is given by -0.9722, indicating a strong negative relationship between a subclasses total static sales and the value of γ which performs best (increases sales the most). The negative sign of the correlation coefficient means for increasing total static sales, the value of γ which is likely to perform best, becomes smaller. The null hypothesis of no linear relationship (no correlation) is tested against the alternative hypothesis of a significant linear relationship (correlation) between total static sales and γ. At a significance level of $\alpha = 0.05$, a t-distribution with $n - 2 = 2$ degrees of freedom gives $t = 5.87$, rejecting the null hypothesis as $p < 0.05$. The correlation coefficient is statistically significant and increasing subclass sizes (in terms of total static sales) are linearly related to a decreasing "chosen γ" values. Concluding the relevance of subclass size when deciding on an appropriate value of γ to use in the dynamic adjustment of size profiles.

CHAPTER 6

Conclusion

Contents

This thesis aims to improve anticipated demand/total sales by dynamically adjusting size profiles as current sales data becomes available throughout the selling season. The main objective is to analyse the effect of dynamic size profile adjustments on total sales for all stores and sizes within the company, as well as unique subsets of stores (and sizes).

This chapter provides a summary of findings in §6.1, recommendations based on results are given in §6.2 and ideas for future work are available in §6.3. The chapter concludes with a thesis summary and the achievement of objectives in §6.4.

6.1 Summary of findings

Size profiles are dynamically adjusted and the effect on total sales are compared to static size profile sales by simulating sales. The simulation model is validated to generate output that is sufficiently close to the Retailer's real system. The effect of dynamic size profile adjustments using various values of a weighting parameter, γ, are analysed for four subclasses. Causation in the change of total sales is established via dynamic adjustment of size profiles.

Through the dynamic adjustment of size profiles, 71–76% of weeks in a product's selling season record an increase in sales, amongst the four subclasses considered in this study. As a result of dynamic size profile adjustments, total sales increase for the two summer and two winter subclasses analysed by a combined 3.11% and 2.72%, respectively. A statistically significant negative correlation between the size of a subclass (in terms of total static sales/anticipated sales) and a value of γ exists.

6.2 Recommendations

It is recommended that the Retailer implement dynamic adjustment of size profiles based on the results obtained in Chapter 5. Pivotal to the success of dynamic size profile adjustments is the choice of a γ value appropriate for the subclass. The magnitude of adjustment experienced by size profiles is established by the value of γ. It is therefore recommended to observe current sales, as the degree of stability is pertinent to an appropriate choice of γ.

Volatility in sales are observed in smaller subclasses and it is recommended that the Retailer use larger values of the weighting parameter, such as $\gamma = 0.7$ or $\gamma = 0.8$. Regardless of subclass size, the use of $\gamma = 0.9$ is not recommended. A value of γ which is too large (*i.e.* $\gamma = 0.9$) generates an insignificant increase in sales, due to a relatively consistent size profile reflective of historical sales (essentially a static size profile).

Based on the results, the Retailer is recommended to use smaller values of γ, such as $\gamma = 0.5$ for larger subclasses. Sales are more stable in larger subclasses and assigning a smaller relative weight to historic sales results in better increases of total sales. However, a decrease in total sales has been observed amongst subclasses where $\gamma = 0.1$. If γ is too small (*i.e.* $\gamma = 0.1$) the resulting adjusted size profile reflects an overcompensation of the current sales performance and is therefore not recommended.

A relationship between subclass size (with regards to anticipated sales/static sales) and the magnitude of adjustment is evident. Inferring the relevance of subclass size in deciding on an appropriate value of γ to use for the dynamic adjustment of size profiles.

6.3 Future work

The scope of this thesis was to analyse the effect of dynamic size profile adjustments on total sales for various values of the weighting parameter, γ. The simulation model was found to accurately represent reality and could be used during further experimentation.

Subclasses analysed in this study have unique characteristics apart from total size, such as number of stores, size of stores, number of styles, frequency and size of styles, to name a few. A beneficial continuation of this study would be to investigate subclass characteristics that have a correlation with the choice of γ values. The identification of correlated characteristics would be of value to the Retailer, assisting the decision making process when determining an appropriate γ value.

A continuation of this study would be to analyse the effect of changing γ during a subclasses selling season. Based on the results obtained in Chapter 5, volatility of current sales at a store results in a sales decrease due to overcompensation from the use of a small γ value. The use of a larger γ value thus performs better in smaller stores. However, as the season progresses and current sales stabilise, the use of a smaller γ value could improve overall performance. Therefore, the effect of changing γ, given the quantity of recorded current sales available, time in the season or based on a forecast of sales, is of interest.

A modification of the simulation model to extend the period of analysis over a few seasons, enabling the analysis of long term effects of dynamic size profile adjustments would provide the Retailer with valuable information regarding sales potential of a subclass. The outcome of long term analysis could be incorporated into the planning process, assisting the Retailer's planners in determining size-mix assortments.

The allocation process forms part of the broader supply chain and distribution network of the Retailer. It would therefore be useful to investigate the downstream effect of dynamic size profile adjustments on the supply chain and distribution network. The Retailer would benefit from such a study which highlights potential sorting, handling and transportation costs from the implementation of dynamic size profile adjustments.

6.4 Thesis summary and achievement of objectives

In Chapter 1 of this thesis, the scope and objectives were explained and Objective 1 was achieved, by providing a description of the allocation process in relation to the Retailer's supply chain and distribution network.

Existing literature on size-mix allocation and a validated simulation model of the Retailer's system based on an underlying regression forecasting model is discussed in Chapter 2, concluding the achievement of Objective 2.

In Chapter 3, collection, cleaning and validation of relevant data to measure the effectiveness of dynamic size profile adjustments and to solve the size-mix allocation, were described in fulfilment of Objective 3. Furthermore, Objective 4 was achieved in this chapter where a simulation model and size-mix allocation were described. A discussion following the development of size profile adjustment is available in Chapter 3, thus achieving Objective 5.

In Chapter 4, verification and validation of the simulation model were done in fulfilment of Objective 6 for each subclass considered in this study and regression assumptions were verified. All four models are valid and accurate representations of reality when a value of the weighting parameter ($\gamma = 1$) specifies size profiles remain static.

The effect of dynamic size profile adjustments were measured in Chapter 5 via the use of validated simulation models, thus fulfilling Objective 7. Experimentation of value variation sensitivity analysis of γ was performed. Concluding causation of dynamic size profile adjustments on the observed increase in total sales and highlighting the importance of an appropriate value of γ. Analysis on the effect of dynamic size profile adjustments on total sales for all stores and sizes within the company, as well as unique subsets of stores (and sizes) indicate a significant improvement in total sales. The Retailer is recommended to incorporate dynamic size profile adjustments into the allocation process.

Finally, in this chapter, findings from the study are summarised and recommendations are made. Ideas for future research are discussed and a summary of the thesis was also provided, concluding the accomplishment of Objective 8.

APPENDIX A

Store categorisation

The effect of dynamic size profile adjustments using various values of weighting parameter, γ are presented in this appendix for each of the four subclasses considered. Stores are categorised according to relative inflows per subclass. Evaluation frameworks presenting the effect dynamic size profile adjustments have on these stores is available in the next sections.

A.1 Subclass S1

Analysis on the effect of dynamic size profile adjustments for Subclass S1 are presented in this section. Sensitivity analysis indicated a value of $\gamma = 0.7$ performed the best, compared to static size profile sales. Stores are grouped into categories relative to their size, with regards to total inflow. A table of categorisation is available in Table 5.3.

An evaluation framework for stores in each category, where S1 "chosen γ" is used to dynamically adjust size profiles is available in §A.1.1. The effect on a company level when size profiles are dynamically adjusted using $\gamma = 0.1$ are presented in §A.1.2, along with evaluation frameworks for stores in Category 2 and 3. Similarly, evaluation frameworks for the analysis of $\gamma = 0.9$ are available in §A.1.3 for the company and stores in Category 2 and 3.

A.1.1 The effect of $\gamma = 0.7$ on S1

Evaluation frameworks for stores in Category 1, 2, 3 and 4 are presented in Figure A.1, A.2, A.3 and A.4 respectively, when size profiles are dynamically adjusted using the "chosen γ", $\gamma = 0.7$. Red diamonds are to the right of corresponding blue squares in each figure, concluding stores in each category record an increase in the amount of sales when size profiles dynamically adjust using $\gamma = 0.7$, compared to sales recorded when size profiles remain static throughout the selling season.

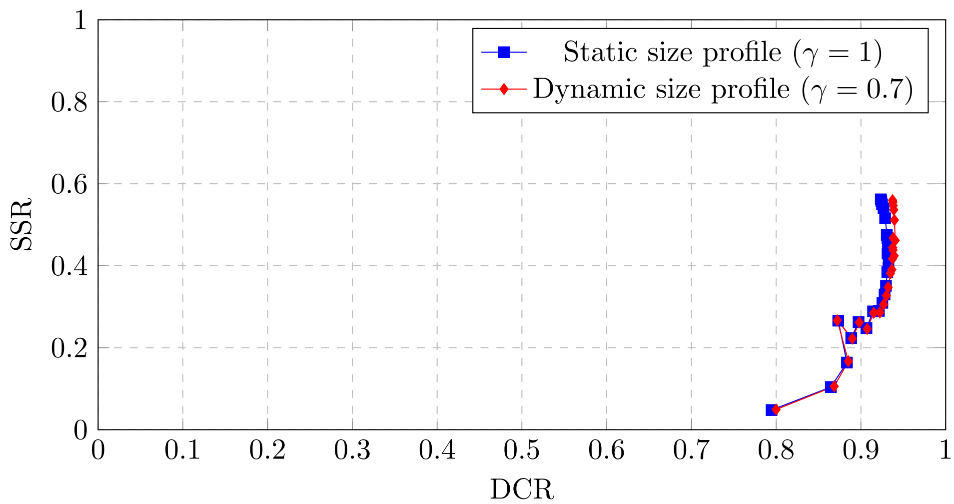

FIGURE A.1: *Subclass S1 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.7$. Considering stores in Category 1.*

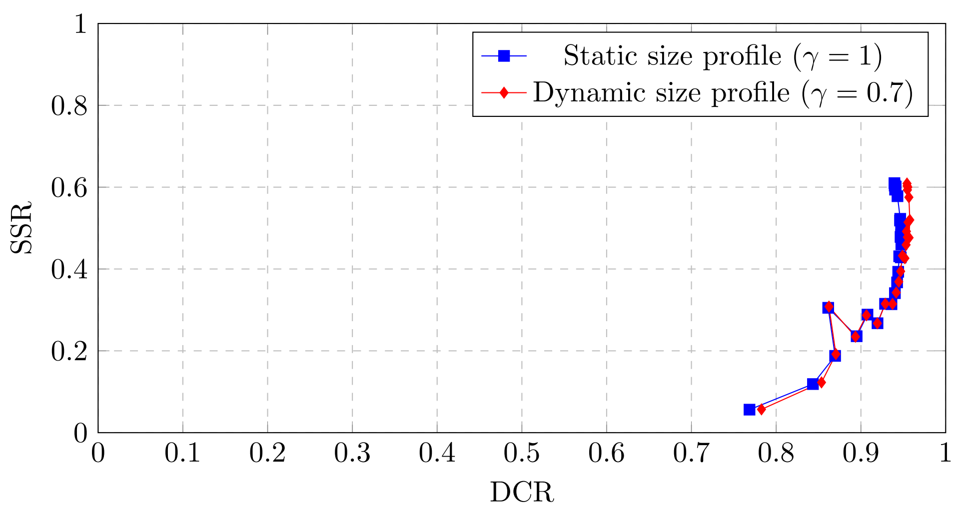

FIGURE A.2: *Subclass S1 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.7$. Considering stores in Category 2.*

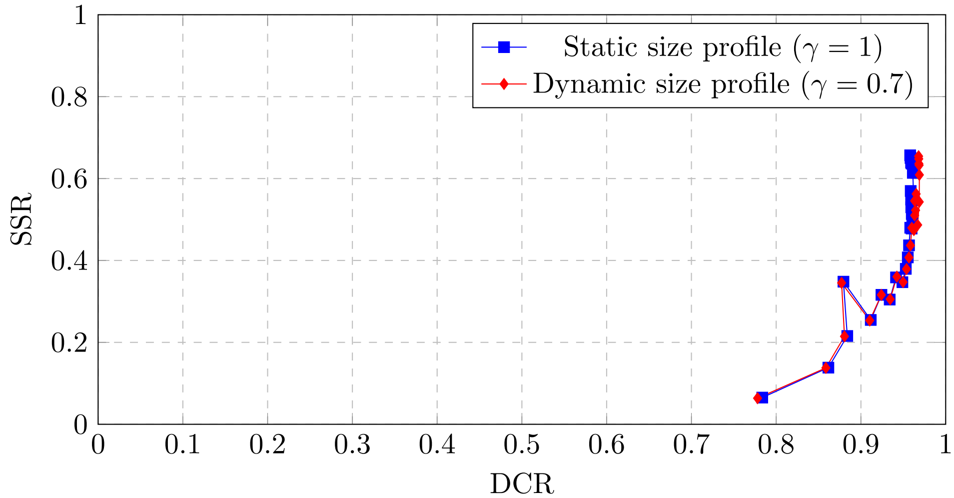

FIGURE A.3: *Subclass S1 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.7$. Considering stores in Category 3.*

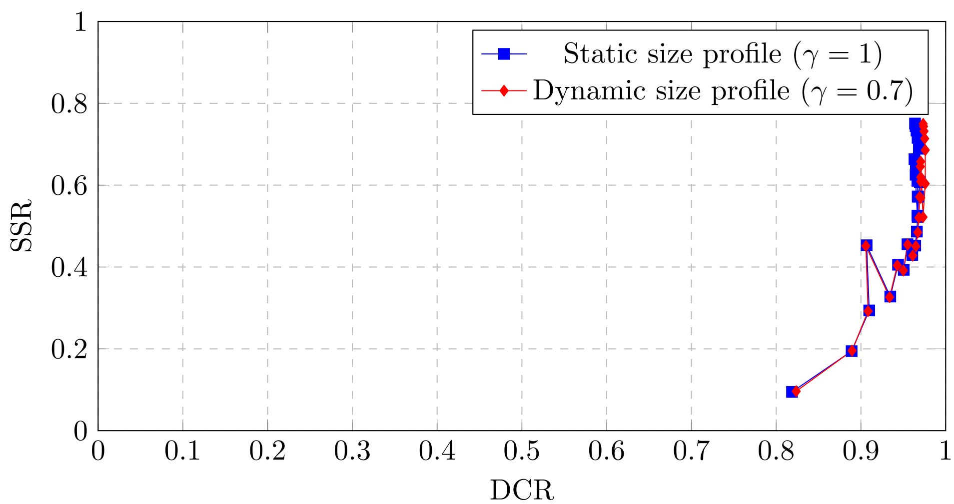

FIGURE A.4: *Subclass S1 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.7$. Considering stores in Category 4.*

A.1.2 The effect of $\gamma = 0.1$ on S1

Evaluation frameworks for all stores in the company are available in Figure A.5. Stores in Category 2 and 3 are presented in FigureA.6 and A.3, respectively. In each figure, red diamonds are to the left and below corresponding blue squares, inferring a decrease in sales when size profiles dynamically adjust using $\gamma = 0.1$, compared to static size profile sales.

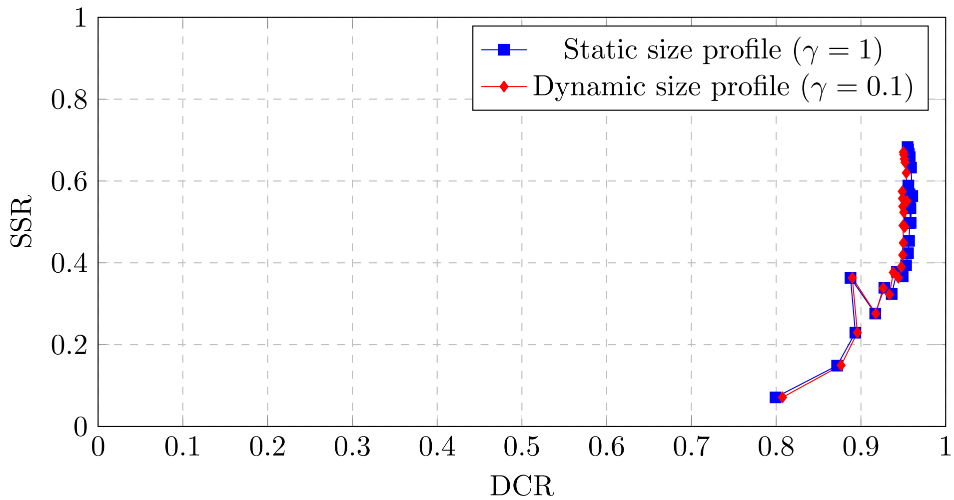

FIGURE A.5: *Subclass S1 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.1$. Considering all stores in the company.*

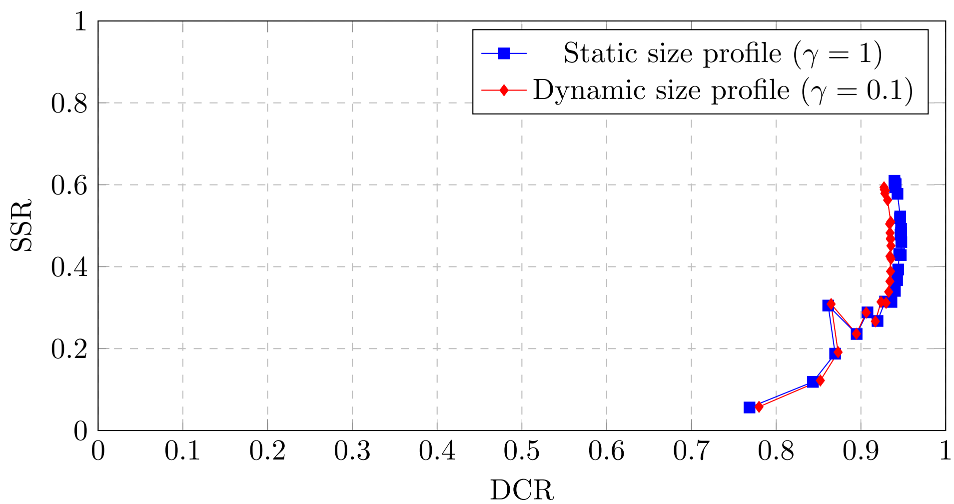

FIGURE A.6: *Subclass S1 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.1$. Considering stores in Category 2.*

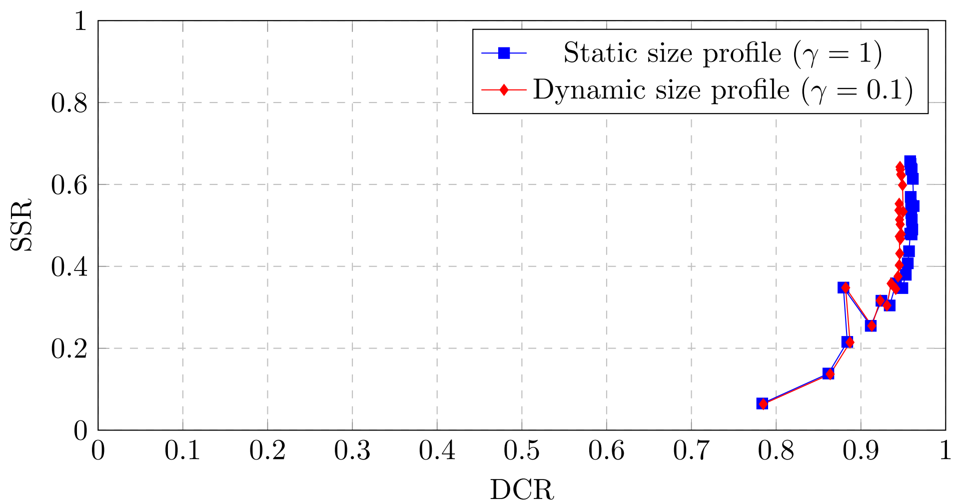

FIGURE A.7: *Subclass S1 evaluation framework of static system performance and dynamic system performance, where* $\gamma = 0.1$. *Considering stores in Category 3.*

A.1.3 The effect of $\gamma = 0.9$ on S1

Figure A.8 depicts an evaluation framework of all stores in the company when size profiles are dynamically adjusted using $\gamma = 0.9$. Similarly, Figures A.9 and A.10 present evaluation frameworks for stores in Category 2 and 3, respectively. Each figure leads to the conclusion that $\gamma = 0.9$, a large weighting parameter; leads to an improvement in sales performance no matter the store size, compared to static size profile sales. However, the performance of $\gamma = 0.9$ is smaller in comparison to dynamically adjusting size profiles using $\gamma = 0.7$. The rightwards shift is smaller when size profiles are dynamically adjusted using $\gamma = 0.9$, than when using $\gamma = 0.7$.

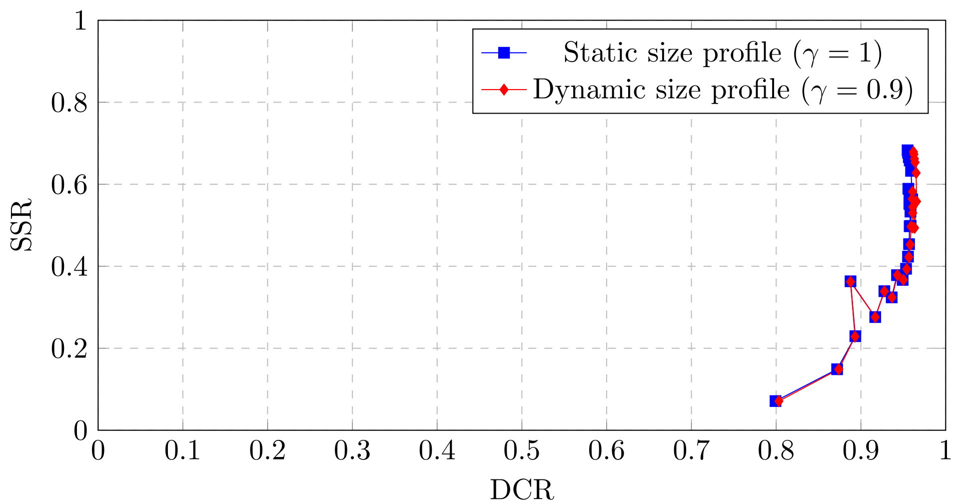

FIGURE A.8: *Subclass S1 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.9$. Considering all stores in the company.*

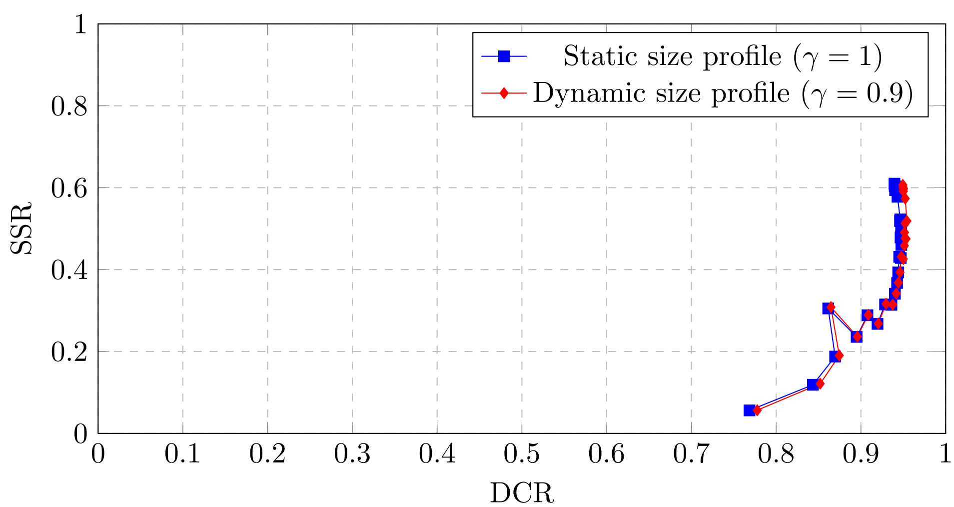

FIGURE A.9: *Subclass S1 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.9$. Considering stores in Category 2.*

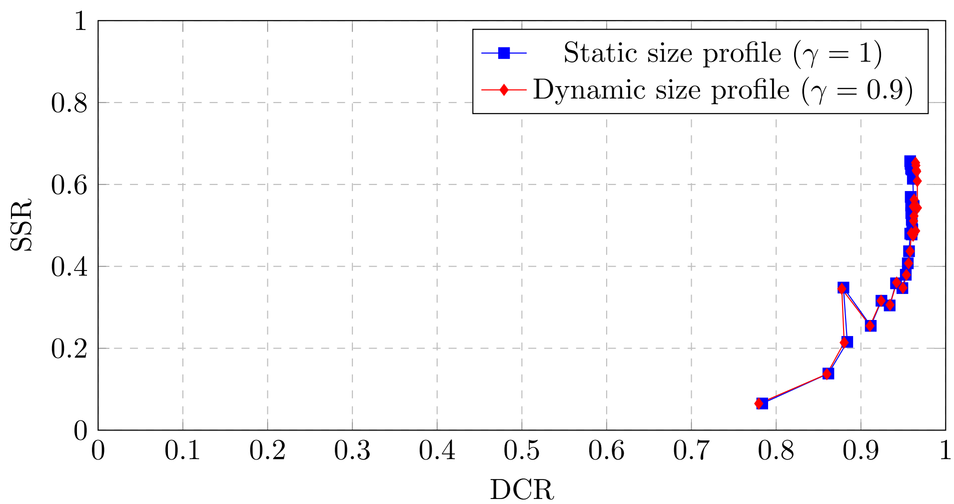

FIGURE A.10: *Subclass S1 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.9$. Considering stores in Category 3.*

A.2 Subclass S2

Evaluation frameworks of the effect on total weekly sales for stores in Subclass S2 are presented in this section. A comparison between dynamic size profile adjustments for different values of γ, are compared to the recorded weekly effect when size profiles remain static. Stores are categorised respective of their inflows, Table 5.6 presents the categorisation of stores in Subclass S2.

The effect on weekly sales for stores in each category when $\gamma = 0.8$ (the "chosen γ") is used to dynamically adjust size profiles are available in §A.2.1. In §A.2.2 and §A.2.3 the effect of extreme weighting parameter values, $\gamma = 0.1$ and $\gamma = 0.9$ are presented, respectively. All stores in the company are analysed, as well as stores in Category 1 and 2 for illustrative purposes as these stores only receive one style inflow and any change is due to random variation.

A.2.1 The effect of $\gamma = 0.8$ on S2

Figures A.11 and A.12 present evaluation frameworks for stores in Category 1 and 2, respectively where random variation is the underlying cause of any effect observed. Stores in Category 3 and 4 receive more than one style inflow and the effect of dynamic size profile adjustments are presented in Figure A.13 and A.14, respectively. It is evident that an improvement in sales performance is recorded when size profiles dynamically adjust using $\gamma = 0.8$, compared to static size profile sales.

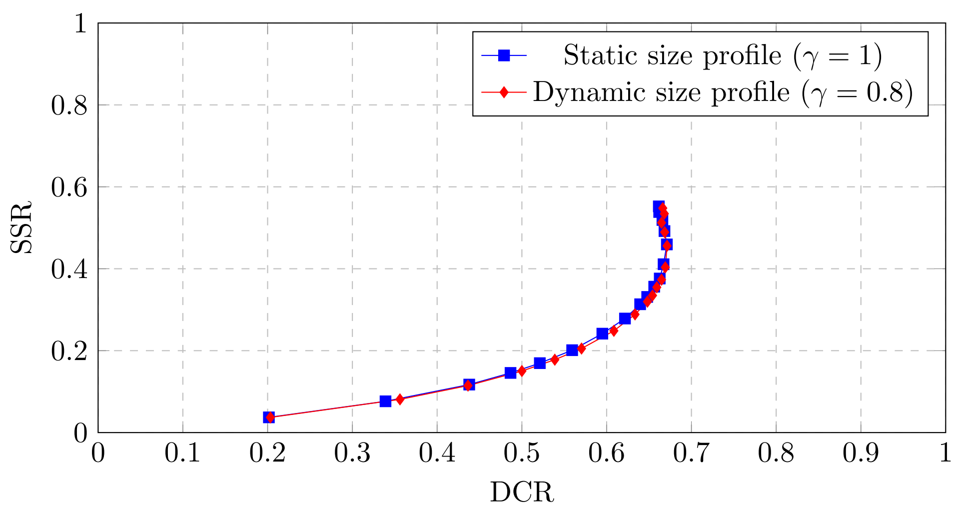

FIGURE A.11: *Subclass S2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.8$. Considering stores in Category 1.*

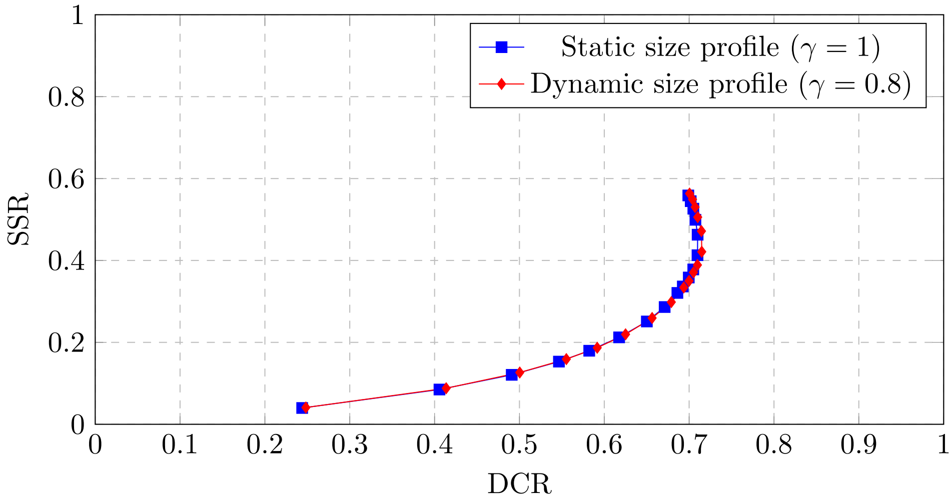

FIGURE A.12: *Subclass S2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.8$. Considering stores in Category 2.*

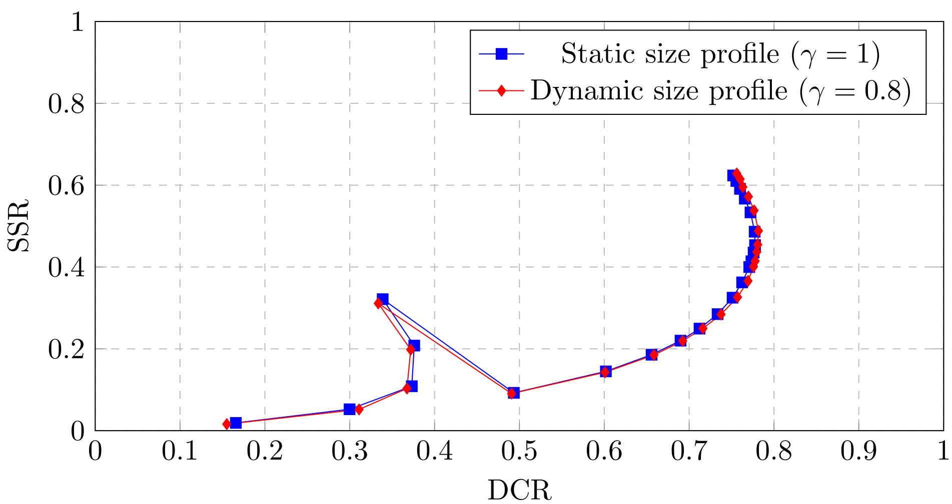

FIGURE A.13: *Subclass S2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.8$. Considering stores Category 3.*

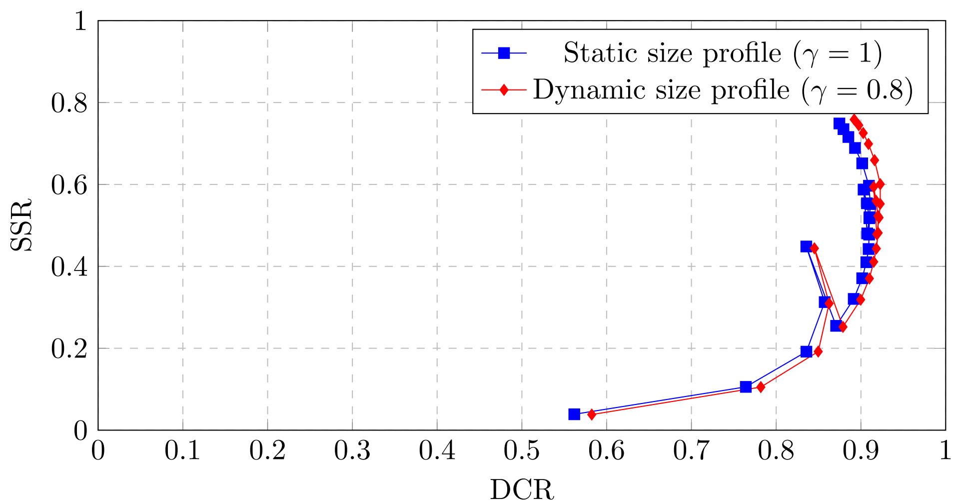

FIGURE A.14: *Subclass S2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.8$. Considering stores in Category 4.*

A.2.2 The effect of $\gamma = 0.1$ on S2

The effect of extreme dynamic size profile adjustments using $\gamma = 0.1$, are presented in this section. On a company level, the effect is available in Figure A.15 it is inferred that a decrease in sales performance will occur for stores in Subclass S2. Figure A.16 and A.17 present the effect for stores in Category 1 and 2, respectively which simply experience random variation and are illustrated simply for comparative purposed.

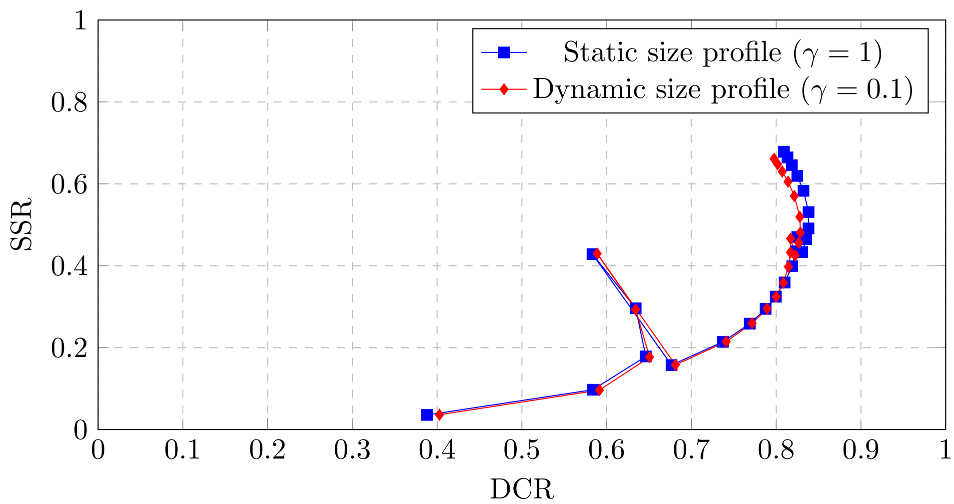

FIGURE A.15: *Subclass S2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.1$. Considering all stores in the company.*

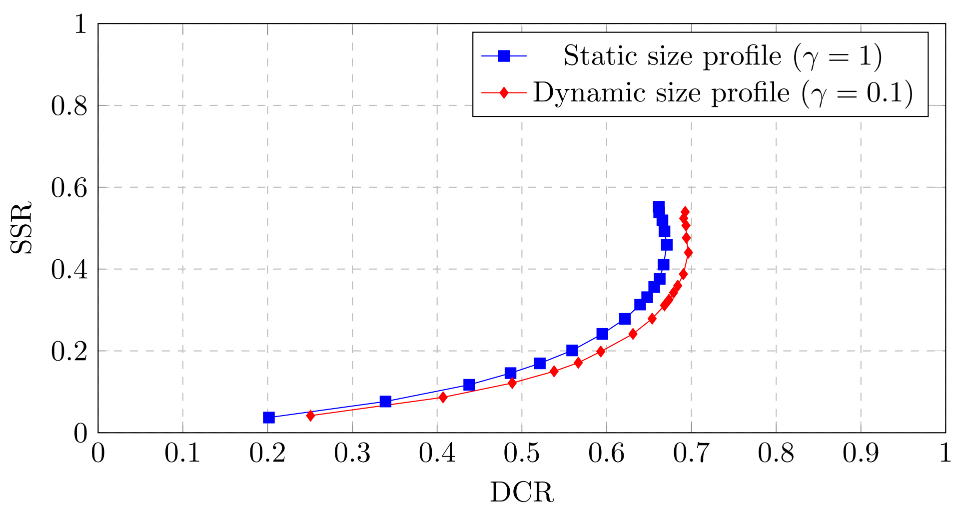

FIGURE A.16: *Subclass S2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.1$. Considering stores in Category 1.*

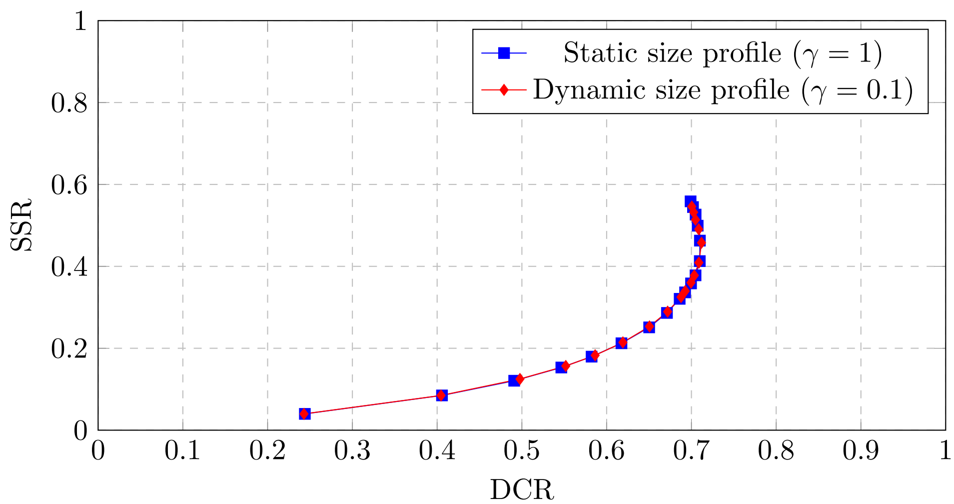

FIGURE A.17: *Subclass S2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.1$. Considering stores in Category 2.*

A.2.3 The effect of $\gamma = 0.9$ on S2

The effect of dynamic size profile adjustments where $\gamma = 0.9$ is used are presented in this section for stores in Subclass S2. Figure A.18 depicts the effect on a company level by considering all stores. A slight increase in inferred, indicated by the marginal rightwards shift of red diamonds compared to corresponding blue squares. Any observed effect in Figures A.19 and A.20 are due to random variation as no size profile adjustments occur for stores in these categories.

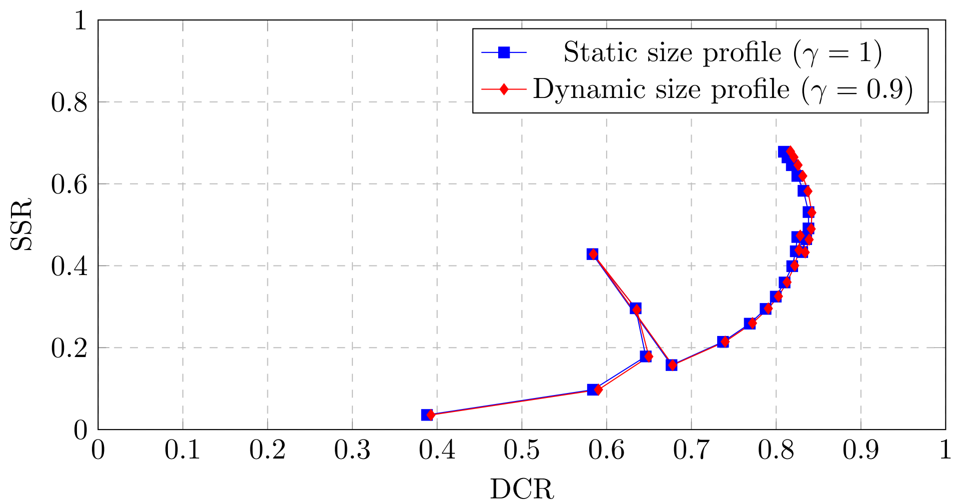

Figure A.18: *Subclass S2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.9$. Considering all stores in the company.*

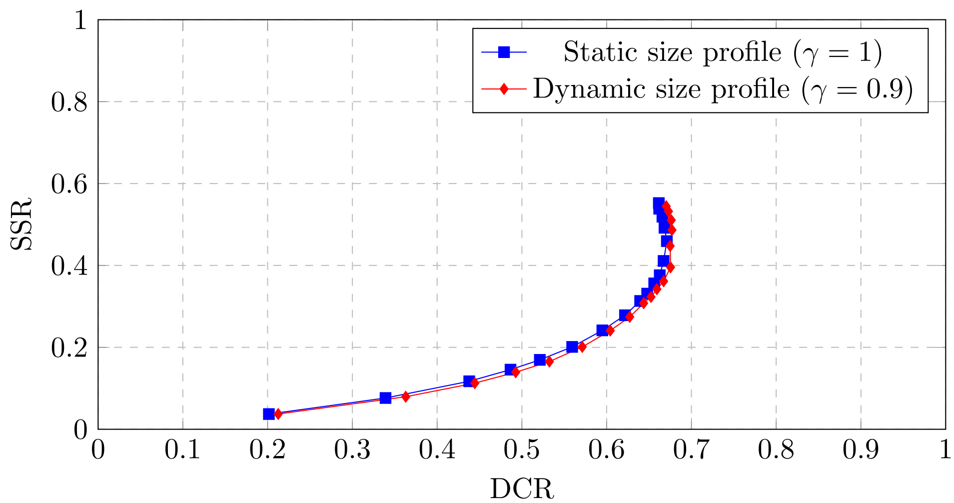

Figure A.19: *Subclass S2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.9$. Considering stores in Category 1.*

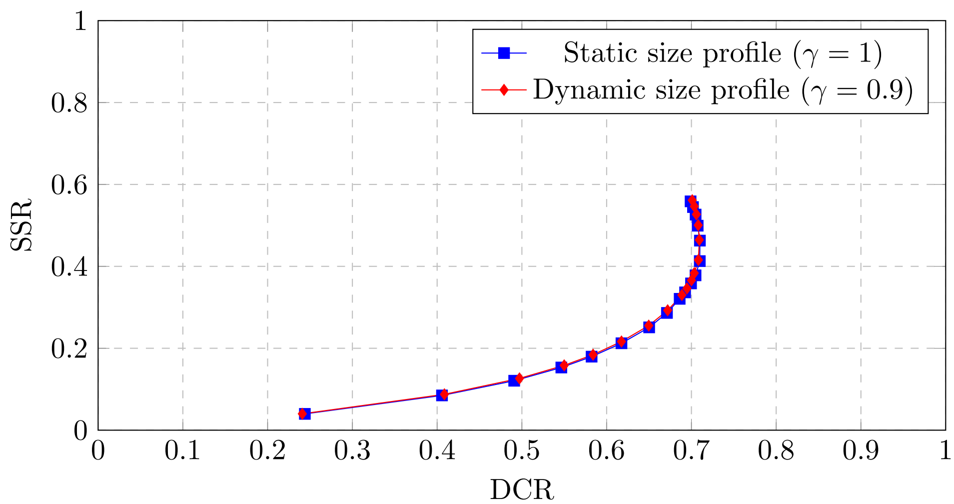

FIGURE A.20: *Subclass S2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.9$. Considering stores in Category 2.*

A.3 Subclass W1

The effect of dynamic size profile adjustments for stores in Subclass W1 are presented in this section. Stores, categorised according to size, are presented in Table 5.9. Total sales increased the most when $\gamma = 0.5$ is used, compared to static size profile sales. An evaluation framework for each of the four categories when $\gamma = 0.5$ is used are presented in §A.3.1. Analysis of the effect on sales where an extreme weighting parameter, $\gamma = 0.1$ and $\gamma = 0.9$ are used is available in §A.3.2 and §A.3.3, respectively.

A.3.1 The effect of $\gamma = 0.5$ on W1

Stores in each category of Subclass W1 report an improvement in sales progress using $\gamma = 0.5$, compared to static size profile sales. The improvement is noted by a rightwards shift along the x-axis and, an upwards shift along the y-axis, indicated by red diamonds (representing the dynamic system) in comparison to corresponding blue squares (representing the static system).

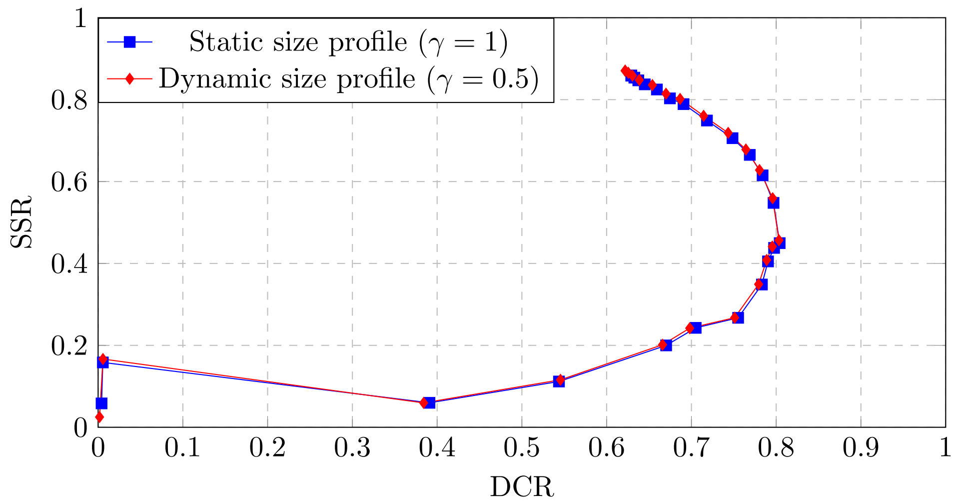

FIGURE A.21: *Subclass W1 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.5$. Considering stores in Category 1.*

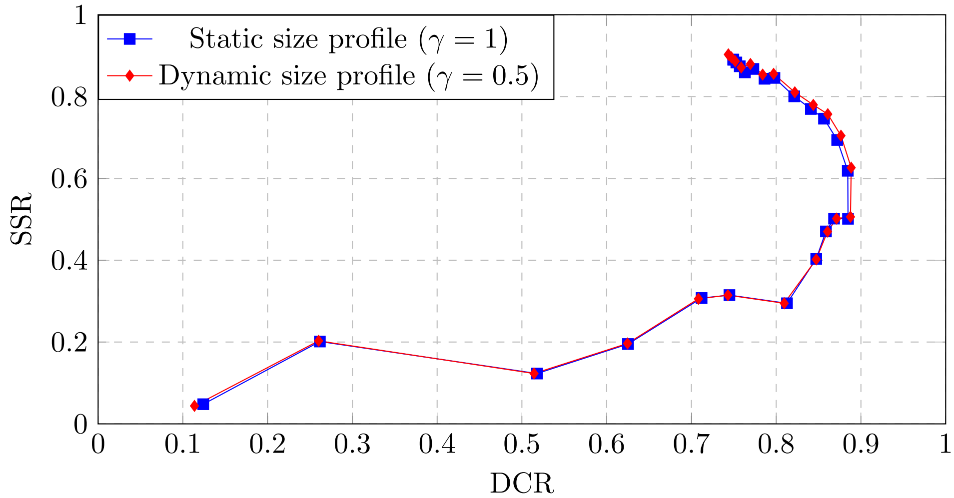

FIGURE A.22: *Subclass W1 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.5$. Considering stores in Category 2.*

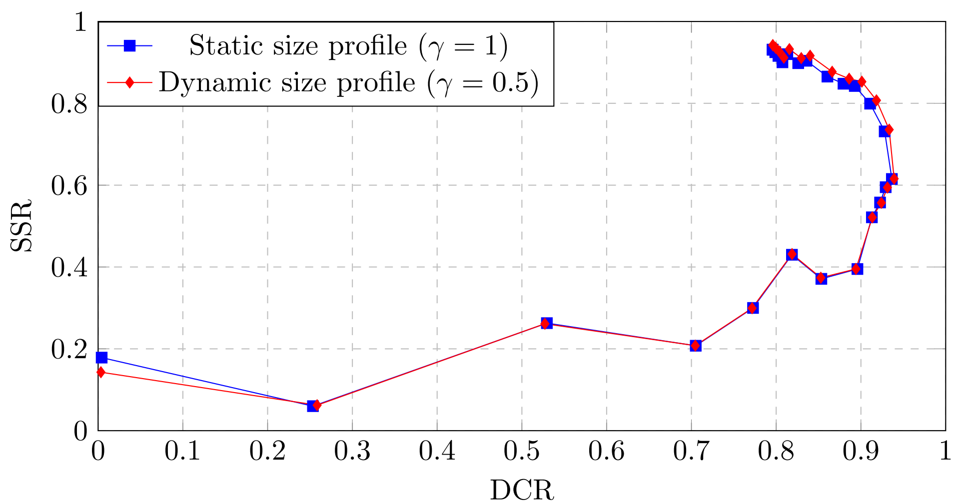

FIGURE A.23: *Subclass W1 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.5$. Considering stores in Category 3.*

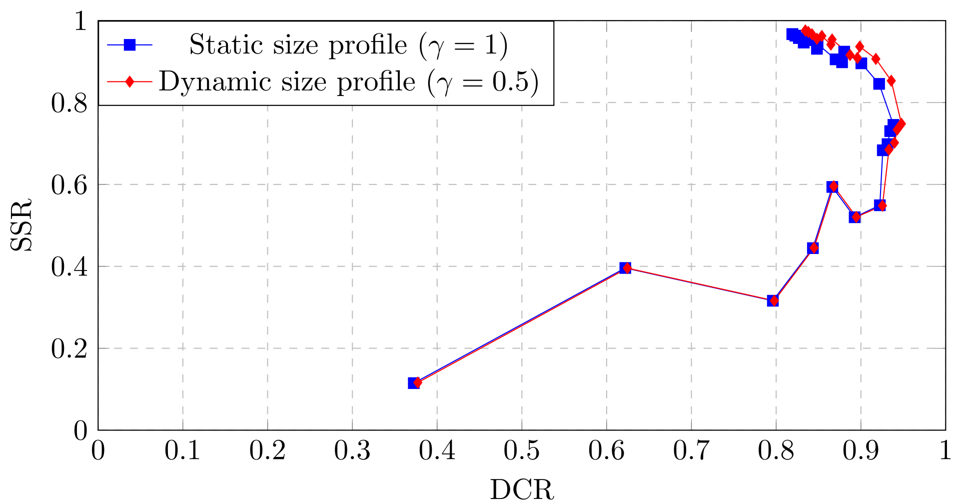

FIGURE A.24: *Subclass W1 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.5$. Considering stores in Category 4.*

A.3.2 The effect of $\gamma = 0.1$ on W1

An evaluation framework of the effect of dynamic size profile adjustments using $\gamma = 0.1$ is presented in Figure A.25 for all stores in the company. There is a slight rightwards shift of red diamonds (representing dynamic system) towards the season's end, inferring that on a company level, sales performance improved compared to the effect when size profiles remain static. Figure A.26 and FigureA.27 present evaluation frameworks of the effect recorded at stores in Category 2 and 3, respectively. A considerable leftwards shift of red diamonds (dynamic) is evident for smaller stores (Category 2), compared to corresponding blue squares (static). A smaller leftwards shift for stores in Category 3 is recorded, inferring the use of $\gamma = 0.1$ for the majority of small stores results in a decrease in sales performance.

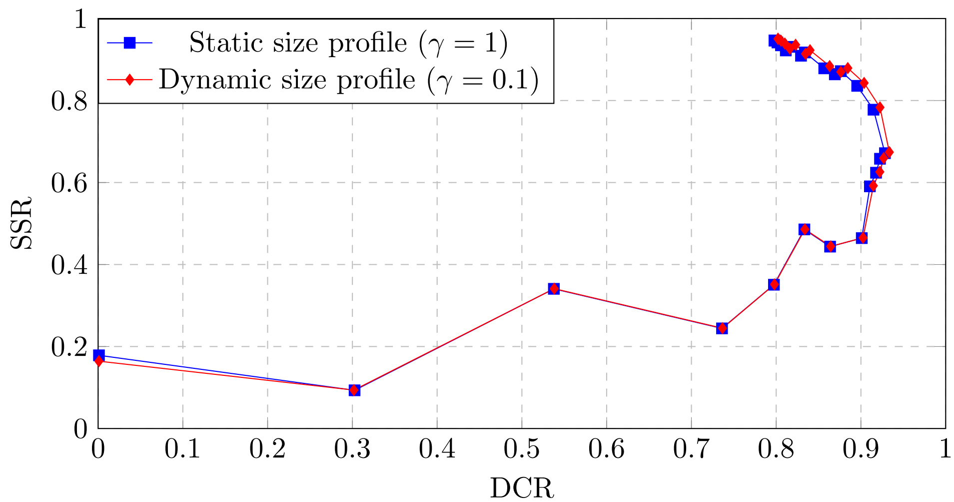

FIGURE A.25: *Subclass W1 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.1$. Considering stores in the company.*

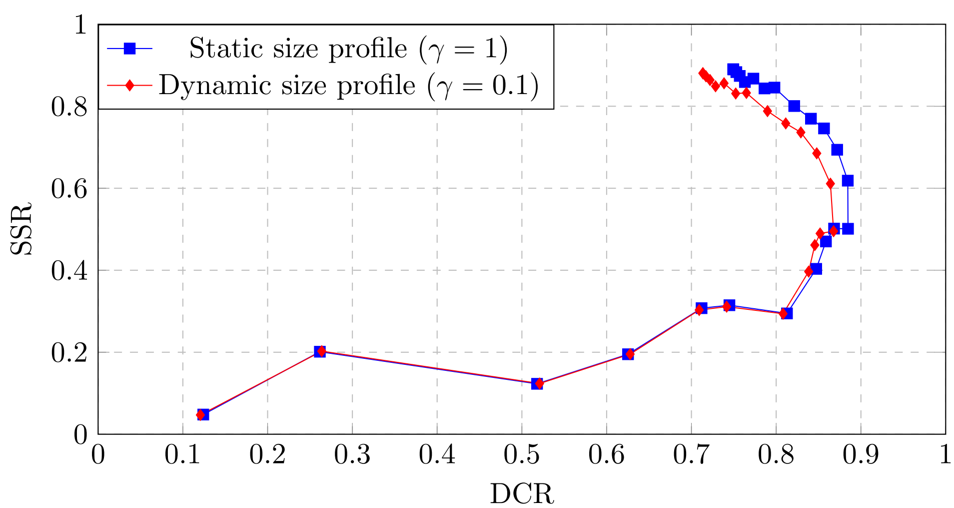

FIGURE A.26: *Subclass W1 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.1$. Considering stores in Category 2.*

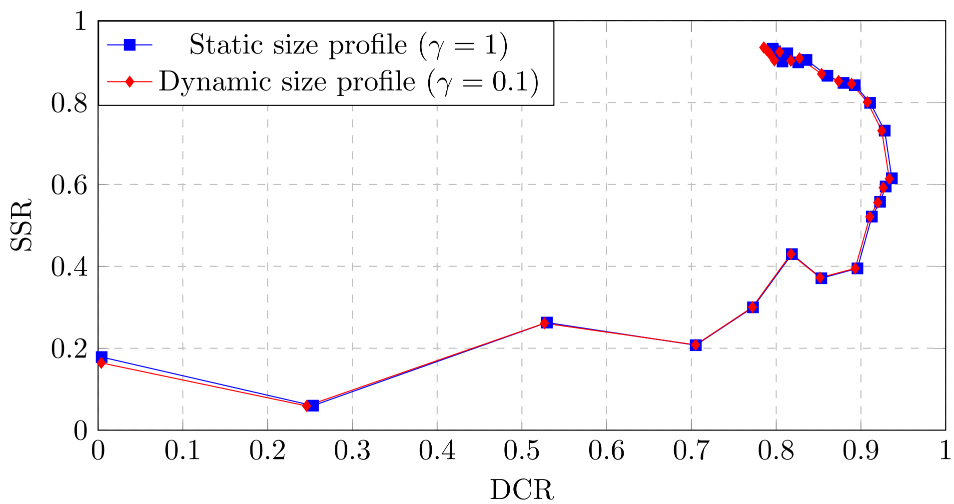

FIGURE A.27: *Subclass W1 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.1$. Considering stores in Category 3.*

A.3.3 The effect of $\gamma = 0.9$ on W1

Figure A.28 presents the effect on weekly sales recorded on a company level at Subclass W1, using $\gamma = 0.9$ in dynamic size profile adjustments (red diamonds), compared to static size profile performance (blue squares). A notable improvement in total sales is recorded, inferring dynamic size profile adjustments using $\gamma = 0.9$ is beneficial for stores in Subclass W1. Furthermore, stores in Category 2 and 3 report a slight rightwards shift of red diamonds compared to static size profile sales. In conclusion, the improvement is inferior compared to using $\gamma = 0.5$ ("chosen γ") for stores in Subclass W1.

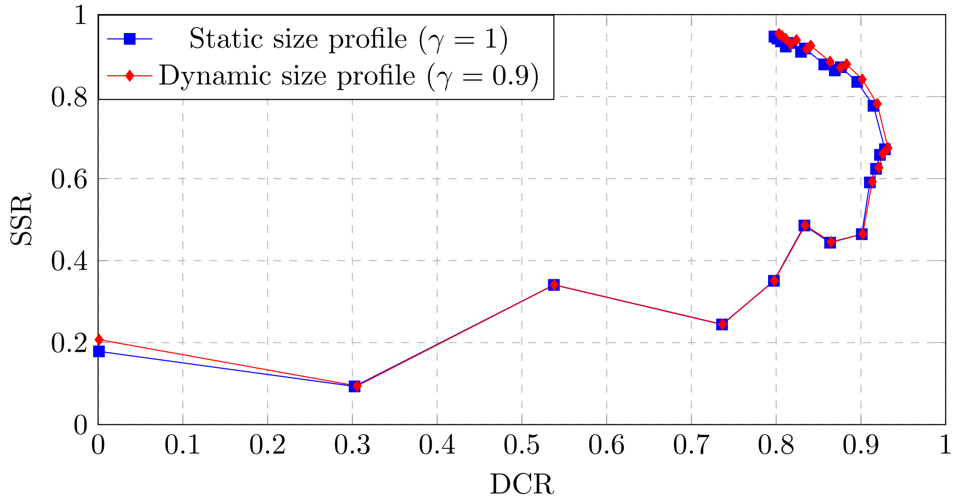

FIGURE A.28: *Subclass W1 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.9$. Considering stores in the company.*

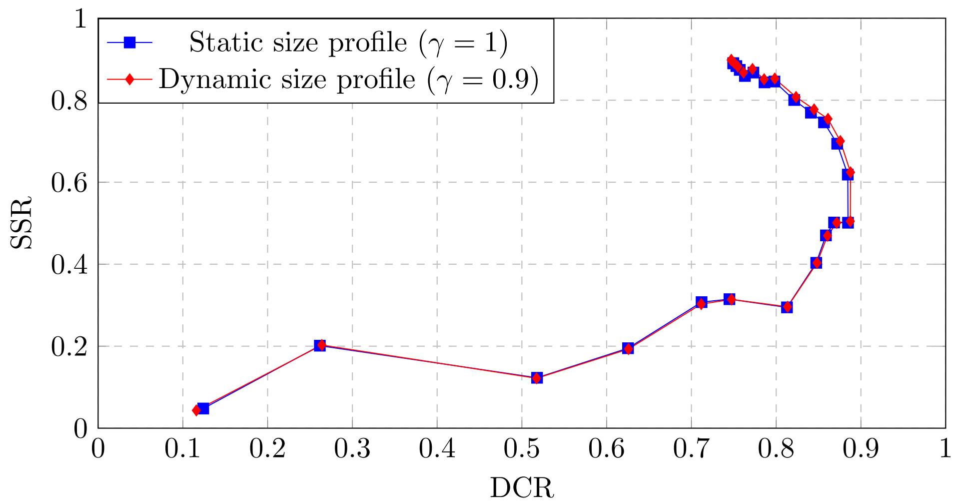

FIGURE A.29: *Subclass W1 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.9$. Considering stores in Category 2.*

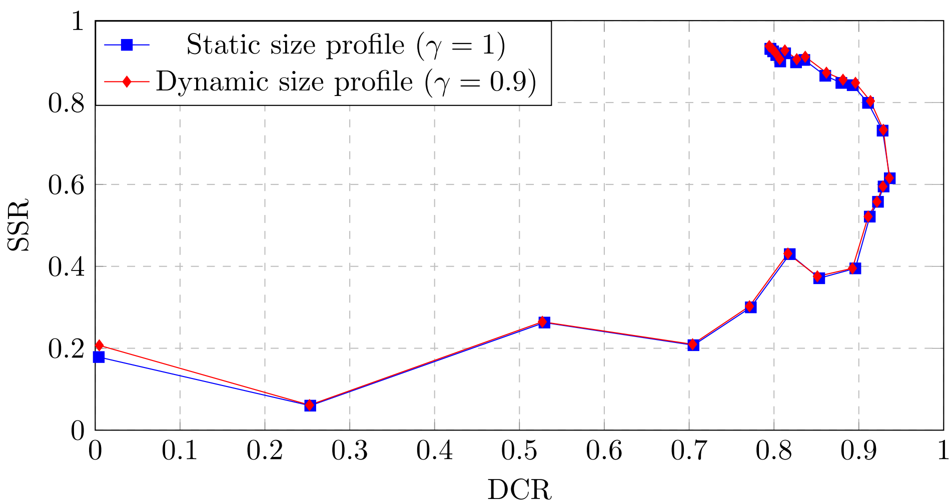

FIGURE A.30: *Subclass W1 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.9$. Considering stores in Category 3.*

A.4 Subclass W2

Stores in Subclass W2 are categorised according to relative inflow amounts, Table 5.12 presents the categorisation of stores analysed. Total sales increase by the largest amount when size profiles dynamically adjust using $\gamma = 0.7$, compared to static size profile sales. The effect recorded for stores in each category are presented in §A.4.1. Size profiles are adjusted using extreme values of weighting parameter, $\gamma = 0.1$ and $\gamma = 0.9$ to analyse the effect on a company, as well as a store level in §A.4.2 and §A.4.3, respectively.

A.4.1 The effect of $\gamma = 0.7$ on W2

Each figure presented in this section indicates a rightwards shift of red diamonds (dynamic system) away from blue squares (static system). It is inferred that dynamic size profile adjustments, using $\gamma = 0.7$ have a positive improvement for stores in each category by increasing sales.

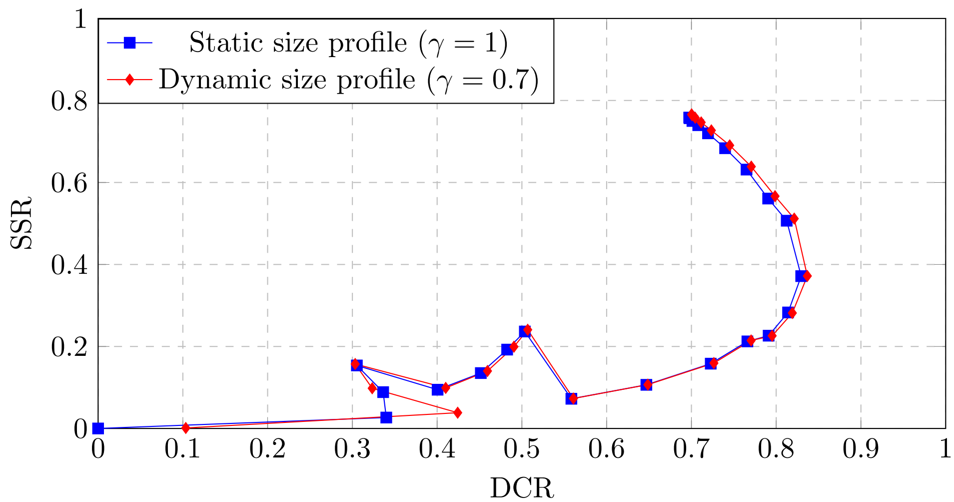

FIGURE A.31: *Subclass W2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.7$. Considering stores in Category 1.*

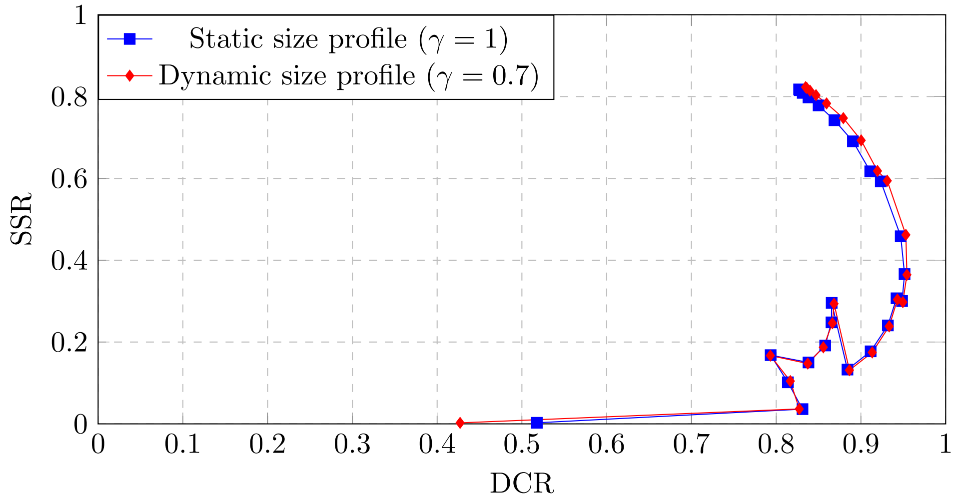

FIGURE A.32: *Subclass W2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.7$. Considering stores in Category 2.*

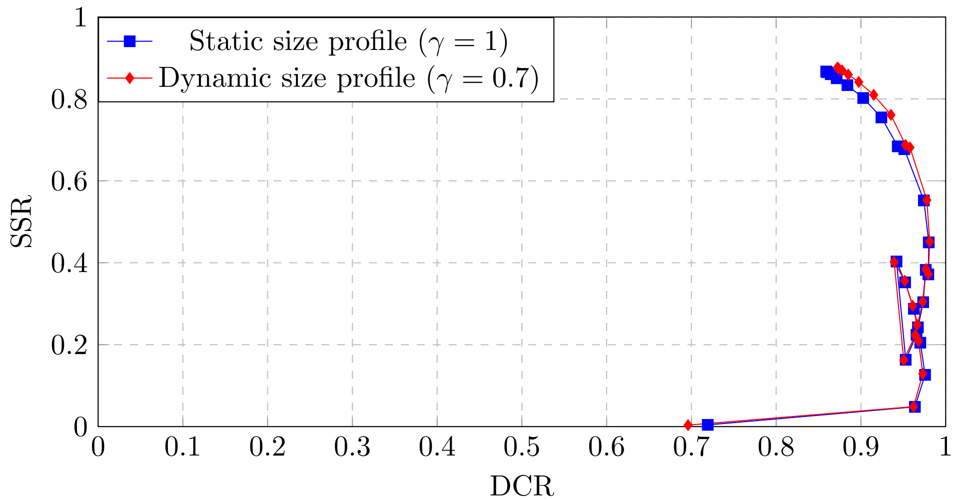

FIGURE A.33: *Subclass W2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.7$. Considering stores in Category 3.*

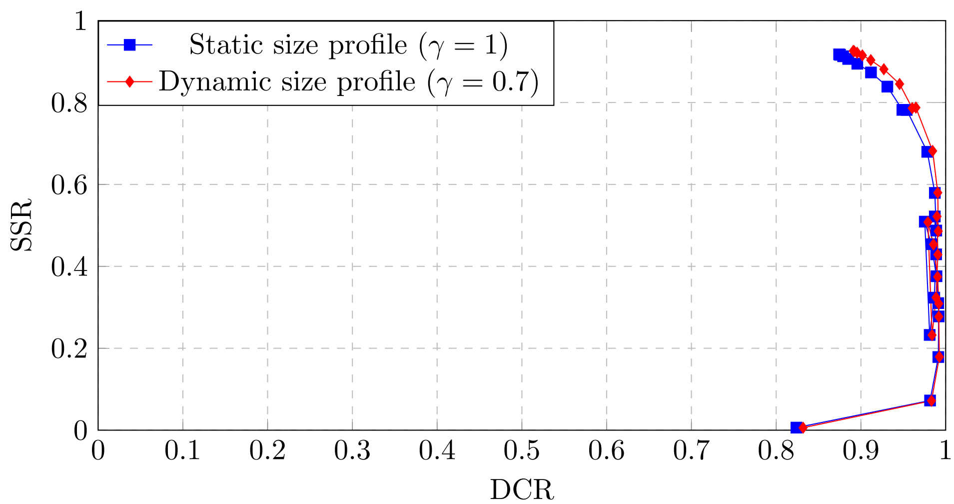

FIGURE A.34: *Subclass W2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.7$. Considering stores in Category 4.*

A.4.2 The effect of $\gamma = 0.1$ on W2

The effect of $\gamma = 0.1$ on a company level is presented in Figure A.35. An evident decrease in sales performance is recorded by the leftwards shift of red diamonds compared to corresponding blue squares. Figure A.36 and Figure A.37 present the effect for stores in Category 2 and 3, respectively. Smaller stores (Category 2) record a larger decrease in sales performance when size profiles dynamically adjust using $\gamma = 0.1$, compared to stores grouped into Category 3. Concluding that smaller stores are better off dynamically adjusting using a larger value of γ as the smaller values (*ie.* $\gamma = 0.1$) result in an overcompensation, leading to decreased sales.

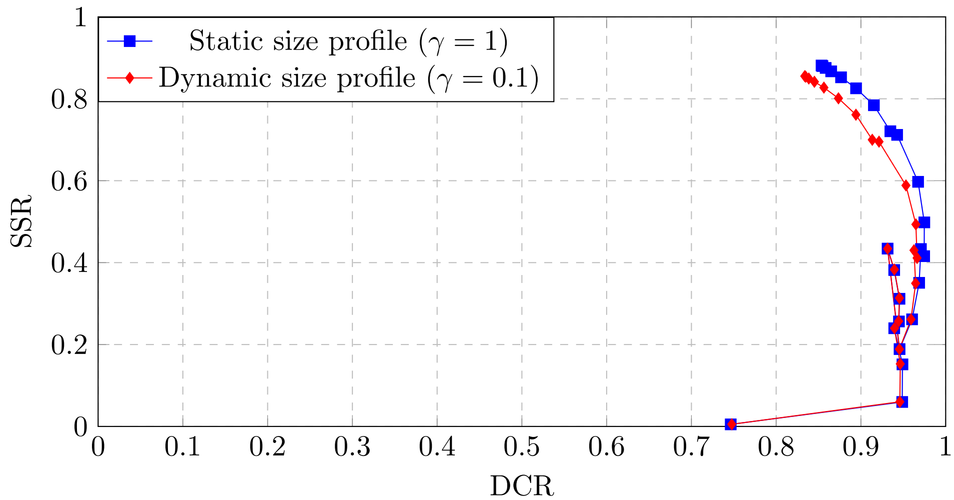

FIGURE A.35: *Subclass W2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.1$. Considering stores in the company.*

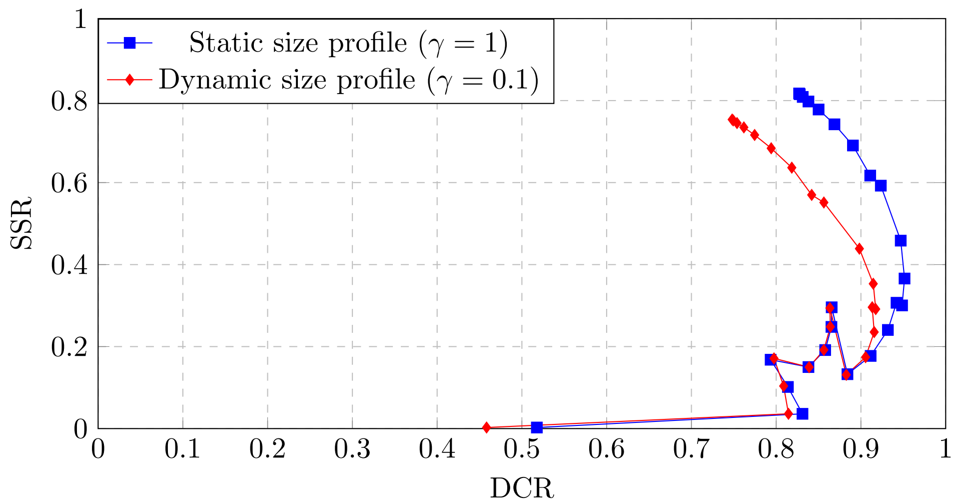

FIGURE A.36: *Subclass W2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.1$. Considering stores in Category 2.*

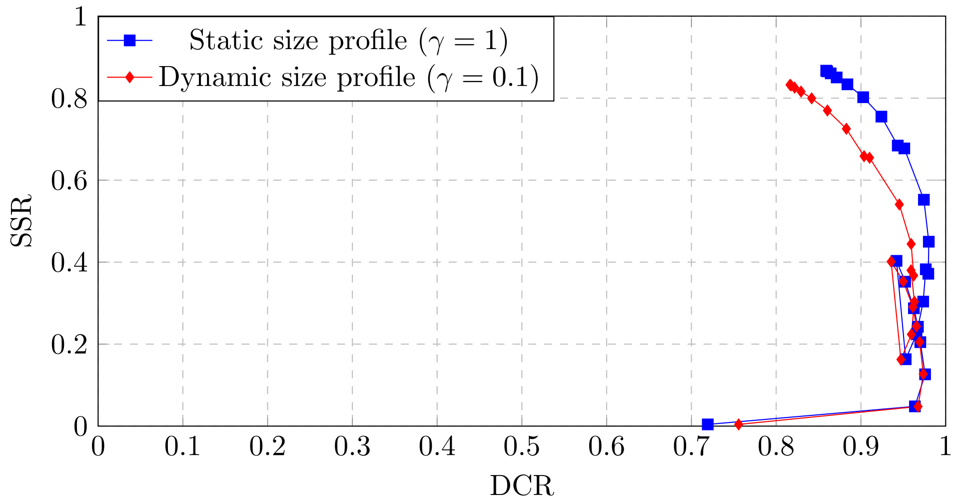

FIGURE A.37: *Subclass W2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.1$. Considering stores in Category 3.*

A.4.3 The effect of $\gamma = 0.9$ on W2

Dynamic size profile adjustments using $\gamma = 0.9$ result in a slight increase in total sales for all stores in the company, compared to static size profile sales. The improvement is indicated in Figure A.38, where a slight rightwards shift of red diamonds, compared to corresponding blue squares are recorded. Figure A.39 and Figure A.40 present the evaluation frameworks for stores in Category 2 and 3, respectively. Each of these figures records a slight improvement in total sales when size profiles are dynamically adjusted using $\gamma = 0.9$, compared to static size profile sales.

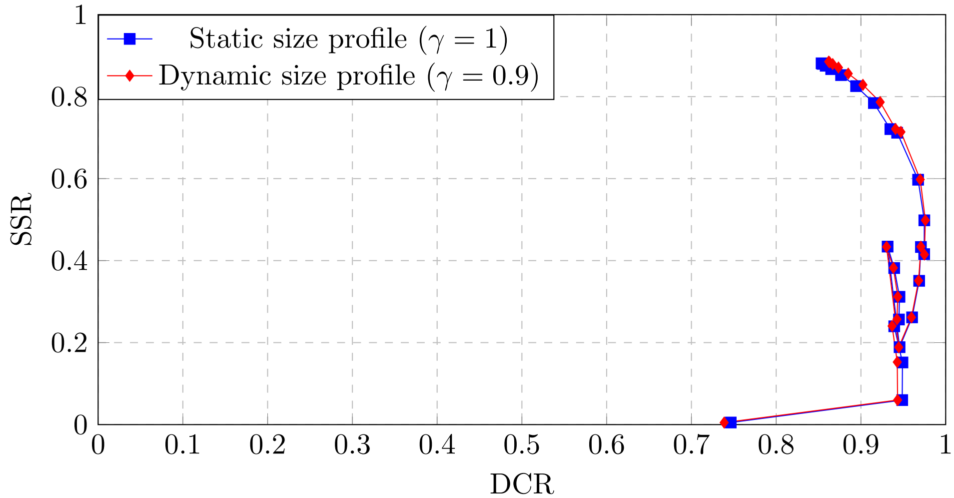

FIGURE A.38: *Subclass W2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.9$. Considering stores in the company.*

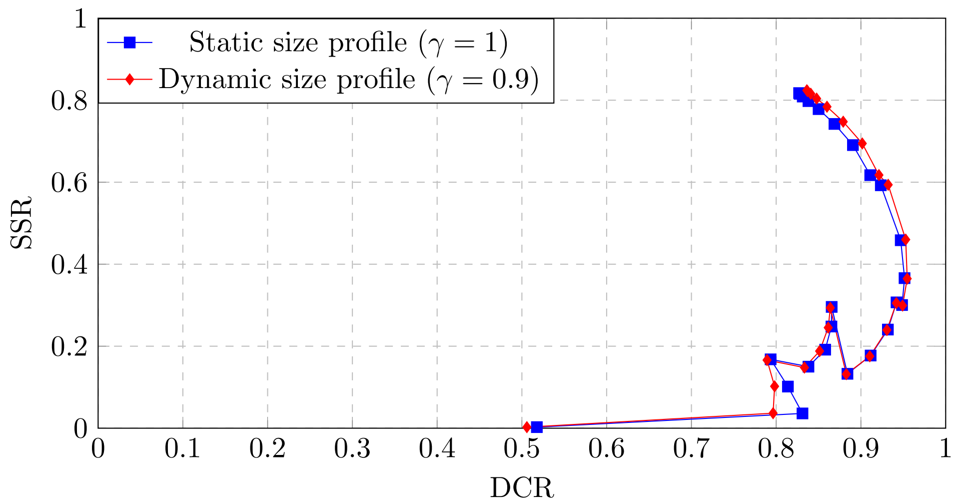

FIGURE A.39: *Subclass W2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.9$. Considering stores in Category 2.*

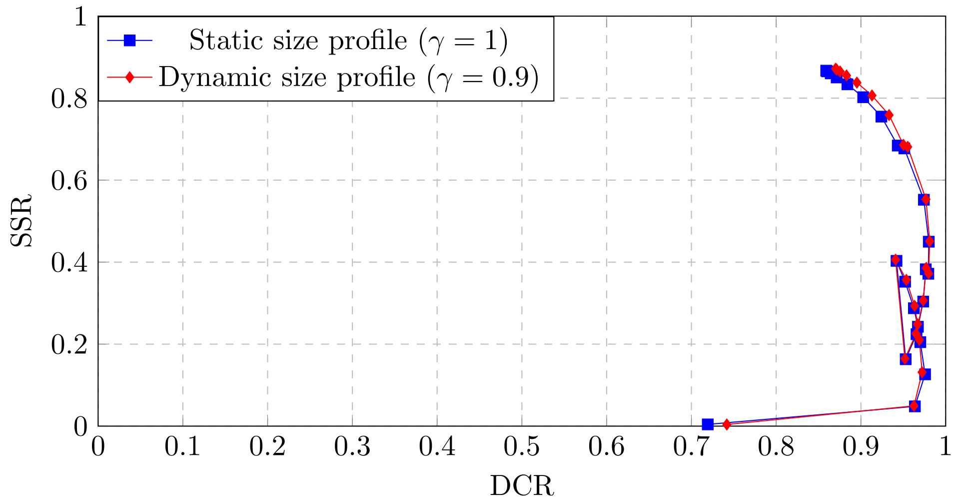

FIGURE A.40: *Subclass W2 evaluation framework of static system performance and dynamic system performance, where $\gamma = 0.9$. Considering stores in Category 3.*

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